Mathematics and Computation in Music
2022 Atlanta, Georgia, USA
June 21-24, 2022

smcm

NSF

Georgia State University | COLLEGE OF ARTS & SCIENCES

School of Music

Contents

1 Welcome to Atlanta

The Eighth International Conference on Mathematics and Computation in Music will be held June 21-24, 2022 at Georgia State University in Atlanta, Georgia, USA.

MCM is the flagship conference of the Society for Mathematics and Computation in Music (SMCM), whose official publication is the Journal of Mathematics and Music (JMM).

MCM 2022 continues the tradition of biennial international conferences of the Society for Mathematics and Computation in Music held on alternating sides of the Atlantic. In this occasion it is hosted by the Georgia State University (GSU).

The conference brings together researchers from around the world who combine mathematics and/or computation with music theory, music analysis, composition and performance. MCM provides a dedicated platform for the communication and exchange of ideas amongst researchers in mathematics, computer science, music theory, composition and performance, musicology and related disciplines.

The disciplines of Mathematics and Music share an intertwined history stretch- ing back more than two and a half millennia. Nowadays computer science points towards new approaches to these disciplines, often with transformative effect.

In addition to the scientific program, there will be concerts open to both conference participants and the general public, plus an outreach afternoon at the Museum of Design Atlanta (MODA).

2 Organization

General Organizing Committee

Mariana Montiel
Department of Mathematics and Statistics, Georgia State University (GSU)

Jeremy Kastine
Department of Mathematics, Life University

Emilio Lluis-Puebla
Departamento de Matemáticas, Facultad de Ciencias, Universidad Nacional Autónoma de México (UNAM)

Guerino Mazzola
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Brent Milam
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Thomas Noll
Escola Superior de Música de Catalunya

Robert Peck
School of Music, Louisiana State University

Robert Schneider
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Moreno Andreatta

Juan Sebastián Arias

Isabel Barbancho

Gilles Baroin

Louis Bigo

Norman Carey

Rodrigo Castro López Vaal

David Clampitt

Richard Cohn

José Miguel Díaz-Báñez

Andrée Ehresmann

Thomas Fiore

Pauxy Gentil Nunes

Francisco Gómez

Julian Hook

Franck Jedrzejewski

Julien Junod

Greta Lanzarotto

Carlos de Lemos Almada

Vicente Liern

Emilio Lluis-Puebla

Maria Mannone

Guerino Mazzola

Brent Milam

Mariana Montiel

Jaime Munárriz

Thoma Noll

Robert Peck

Alexandre Popoff

Brian Martínez-Rodríguez

Lauren Ruth

Robert Schneider

William Sethares

Julius Smith

Dmitri Tymoczko

Jason Yust

Local Organizing Committee

Rose Friend

Junwoon Seo

Juliana Spector

Sribhuvan Yellu

3 Conference Schedule

Registration will be available throughout the conference

The talks will take place from 9am-2pm at the Lecture Hall, Event Center, 25 Park Place 2cd floor, Atlanta, GA 30303

Plenary talks, panels and concerts will take place from 5-8pm at the Florence Kopleff Recital Hall, 15 Gilmer SE, Atlanta, GA 30303

3.1 Tuesday, June 21, 2022

Lecture Hall, Event Center, 25 Park Place 2cd floor

9:00-9:30: ***Opening Inaugural Session***:

Dr. Sara Rosen, Dean of the College of Arts and Sciences, GSU

Dr. Wade Weast, Dean of the College of the Arts.

Dr. Moreno Andreatta, Vice president and acting president of the Society for Mathematics and Computation in Music, University of Strasbourg and IRCAM, France

Dr. Jason Yust, Chief editor of the *Journal of Mathematics and Music*, School of Music, Boston University

Dr. Mariana Montiel, Chair of the Organizing and Scientific Committees, Department of Mathematics and Statistics, GSU

Mathematical Scale and Rhythm Theory: Combinatorial, Graph Theoretic, Group Theoretic, and Transformational Approaches

Chairs: *Octavio Alberto Agustín-Aquino & Brent Milam*

9:35-10:05*: A Set-Theoretic Model of Metric Relations*

Richard Cohn

10:10-10:40 ***New insights on diatonicity and majorness***

Franck Jedrzejewski

10:45-11:15 ***Parsimonious Graphs for Selected Heptatonic and Pentatonic Scales***

Luis Nuño

11:15-11:30 *Coffee Break*

11:35-12:05 ***An interactive tool for composing (with) automorphisms in the colored Cube Dance***

Alexandre Popoff, Corentin Guichaoua & Moreno Andreatta

12:10-12:40 ***Combinatorial Spaces***

Robert Peck

12:45-1:15 ***Euler's "Tentamen"***

Sonia Cannas

1:20-2:00

Poster session[*]

1. ***Persistent Homology on Musical Bars***
   Victoria Callet

2. ***Identifying Metric Types with Optimized DFT and Autocorrelation Models***
   Matthew Chiu & Jason Yust

3. ***Geometry of Music in the Parabola***
   Edgar Delgado

4. ***midiVERTO: A Web Application to Visualize Tonality in Real Time ,***
   Daniel Harasim, Giovanne Affatato & Fabian Moss

5. ***Quantum-Musical Explorations on Zn***
   Thomas Noll

6. ***The Mystery of Anatol Vieru's periodic sequences unveiled***
   Luisa Fiorot, Alberto Tonolo & Riccardo Giblas

7. ***Benford's Law and Music***
   Sybil Prince, Brian Wickman, Jack Null, Eric Gazin

8. ***Altered Chord Alternatives***
   Lauren Ruth

9. ***Information Synthesis of Time-Geometry QCurve for Music Retrieval***
   Shanon Steinmetz & Ellen Gethner

10. ***Investigating Style with Scale Embeddings***
    Matt Chiu

[*] ***The Posters will be exhibited throughout the entire conference***

2:00 pm–5:00 pm: *Lunch and free time to visit Atlanta attractions*

There are a myriad of restaurants of every type of food imaginable in the immediate surrounding area, several with outside seating.

Evening Session

Florence Kopleff Recital Hall, 15 Gilmer SE

5:00 pm: **Panel**

"*What is our shared baseline knowledge set*"

moderated by ***James Hughes*** with the participation of: ***Richard Cohn, Francisco Gómez, Julian Hook*** and ***Thomas Noll***

6:30 pm **Concert**

"Music and maths: the geometric match"

Emmanuel Amiot, Moreno Andreatta and Gilles Baroin.

3.2 Wednesday, June 22, 2022

Categorical and Algebraic Approaches to Music

Chairs: *Emilio Lluis-Puebla* & *Mariana Montiel*

9:00-9:30 ***A Projection-Oriented Mathematical Model for Second-Species Counterpoint***

Octavio Alberto Agustín-Aquino & Guerino Mazzola

9:35-10:05 ***When Virtual Reality helps fathom Mathemusical Hyperdimensional Models***

Giles Baroin

10:10-10:40 ***SUM Classes and Quotient Generalized Interval Systems***

David Orvek & David Clampitt

10:45-11:15 ***Extended Vuza canons***

Greta Lanzarotto & Ludovico Pernazza

11:15-11:30 *Coffee Break*

11:35-12:50 General Meeting of the SMCM

Due to the retirement of Guerino Mazzola as president of the Society for Mathematics and Computation in Music, we take advantage of this gathering to make plans for the future of our professional organization.

12:55-1:25 ***Some mathematical and computational relations between timbre and color***

Maria Mannone & Juan Sebastián Arias-Valero

1:30-2:00 ***Pairwise Well-Formed Modes as Transformations***

Thomas Noll & David Clampitt

__2:00-3:00__ ***Editorial Board Meeting of the Journal of Mathematics and Music*** *Lunch will be served for members of the board.*

2:00 pm–5:00 pm: *Lunch and free time to visit Atlanta attractions*

There are a myriad of restaurants every type of food imaginable in the immediate surrounding area, several with outside seating.

__Evening Session__

Florence Kopleff Recital Hall, 15 Gilmer SE

5:00 pm **Plenary Talk:**

"*A Cornucopia of Musical Spaces*"

Julian Hook

6:30 pm **Concert**

"Positive and Negative Spaces"

Terminus Ensemble of Contemporary Music

3.3 Thursday, June 23, 2022

Applications of Mathematics to Musical Analysis

Chairs: *David Clampitt & Robert Peck*

9:00-9:30 ***Mathematical Morphology Operators for Harmonic Analysis***

Gonzalo Romero-García, Isabelle Bloch & Carlos Agon

9:35-10:05 ***Computational Analysis of Musical Structures based on Morphological Filters***

Paul Lascabettes, Paul, Carlo Agon, Moreno Andreatta, and Isabelle Bloch

10:10-10:40 ***Non-Spectral Transposition-Invariant Information in Pitch-Class Sets and Distributions***

Jason Yust & Emmanuel Amiot

10:45- 11:15 ***Tetrachordal Folding Operations***

Jason Yust

Mathematical Techniques and Microtonality

Chair: *Brent Milam*

11:20-11:50 ***Continuous Chromagrams and Pseudometric Spaces of Sound Spectra***

Jordan Lenchitz & Anthony Coniglio

11:55-12:25 ***N2D3P9***

Dave Keenan & Douglas Blumeyer

12:30-1:00 ***Performing Easley Blackwood's Twelve Microtonal Etudes: An Open-Source Software Development Project***

Richard Leinecker & William Ayers

1:30-4:30pm *Math+Music@MODA*

Outreach afternoon at the Museum of Design Atlanta (MODA)
1315 Peachtree St. NE Atlanta, GA 30309

- *Jeremy Kastine, (Organizer) "**Can You Canon?**"*
- *Gilles Baroin, "**MatheMusical VR Movies and Interactive Models**"*
- *Paco Gómez, "**Matherhythm or rhythm is a killer**"*
- *Maria Mannone, "**The CubeHarmonic**"*
- *Thomas Noll, "**The Collective Public Fourier Performance**"*
- *Luis Nuño, "**The Harmonic Wheel**"*

Evening Session

Florence Kopleff Recital Hall, 15 Gilmer SE

A Homage to Jack Douthett

5:30 pm Plenary Session ***with the participation of:***

Emmanuel Amiot, David Clampitt, Richard Cohn, Julian Hook, Richard Krantz and Thomas Noll.

7:00 pm Concert ***with the participation of:***

Octavio Agustín-Aquino, Emmanuel Amiot, Moreno Andreatta, Giles Baroin, Leah Frederick, Julian Hook, Thomas Noll, and Emilio Lluis-Puebla accompanying soprano Juliana Spector.

3.4 Friday, June 24, 2022

<u>Algorithms and Modeling for Music and Music-Related Phenomena</u>

<u>Chairs:</u> *Jeremy Kastine & Mariana Montiel*

<u>9:00-9:30</u> ***Spline Modeling of Audio Signals with Cycle Interpolation***

Matthew Klassen

<u>9:35-10:05</u> ***Transposition and Time-Scaling Invariant Algorithm for Detecting Repeated Patterns in Polyphonic Music***

Antti Laaksonen, Kiell Lemström & Otso Björklund

<u>10:10-10:40</u> ***On the Memory Usage of the SIA Algorithm Family for Symbolic Music Pattern Discovery***

Antti Laaksonen, Kiell Lemström

<u>10:55-11:15</u> ***A proposal to compare the similarity between musical products. One more step for automated plagiarism detection?***

Aarón López-García, Brian Martínez-Rodríguez & Vicente Liern

<u>11:15-11:30</u> ***Coffee Break***

<u>11:35-12:05</u> ***A new fitness function for evolutionary music composition***

Brian Martínez-Rodríguez

<u>12:10-12:40</u> A mathematical model of tonal function: voice leadings

Francisco Gómez & Isaac del Pozo

<u>12:45-1:15</u> ***A mathematical model of tonal function: modulation***

Francisco Gómez & Isaac del Pozo

<u>1:20-1:50</u> ***Hypercube + Rubik's Cube + Music = HyperCubeHarmonic***

Maria Mannone, Takashi Yoshino, Pascal Chiu, Yoshifumi Kitamura

2:30 pm–5:00 pm: *Lunch and free time to visit Atlanta attractions*

There are a myriad of restaurants every type of food imaginable in the immediate surrounding area, several with outside seating.

Evening Session

Florence Kopleff Recital Hall, 15 Gilmer SE

5:00 pm: Panel

"*What are the boldest expectations in Mathematical Music Theory*"

moderated by ***Thomas Noll*** with the participation of ***Guerino Mazzola*** **(***via Zoom***)**, ***Octavio A. Agustín-Aquino, Emmanuel Amiot, David Clampitt, Tom Fiore. Mariana Montiel,*** and ***Jason Yust***

6:30 pm Concert

"Rachmaninoff's Faust Piano Sonata Op. 28" (context and performance)

Emilio Lluis-Puebla.

8:30 pm ***Gala Dinner***

4.1 Abstracts for Talks

Mathematical Scale and Rhythm Theory: Combinatorial, Graph Theoretic, Group Theoretic, and Transformational Approaches

1. ***A Set-Theoretic Model of Metric Relations***

Richard Cohn

A set-theoretic of model of musical meter is formalized, building up from time points to pulses to meters to metric relations. The model of metric relations formalizes work on metric dissonances, and refines the displacement/grouping taxonomy under current usage.

Keywords*: Musical meter, set theory, metric dissonance, syncopation, hemiola, polymeter*

2. ***New insights on diatonicity and majorness***

Franck Jedrzejewski

This article contributes to the study of diatonicity in tunings with N equal divisions of the octave. The new definition we are proposing of a diatonic scale is based on two concepts. The first is well known since it concerns generated scales studied by Norman Carey and David Clampitt. The second is much less known. It is based on the sets of progressive transposition introduced by the French composer Alain Louvier. From these two concepts, we formulate a new definition of microdiatonic scales which is entirely characterized by two fundamental parameters. Then we define the majorness of a scale, by introducing an interval equivalent of the tritone by using limited transposition sets. We conclude this article by observing that under these conditions diatonicity and majorness are two different characters which do not necessarily exist in all tunings.

Keywords*: Diatonic scales, Diatonicity, Diatonic Theory, EDO, Major scales, Relative Minor Scales, Microintervals*

3. ***Parsimonious Graphs for Selected Heptatonic and Pentatonic Scales***

Luis Nuño

Tonal music is based on major, melodic and harmonic minor scales. In some cases, the harmonic major scale is also used. In this paper, four additional heptatonic scale types, derived from them, are considered. The harmonic characteristics of these eight scale types are analyzed by the trichord- and tetra-chord-type vectors, which list, respectively, the number of times each trichord and tetrachord type is contained in a set type. Then, a novel parsimonious graph is provided, called 7-Cyclops, which relate those scales by single-semitonal transformations. On the other hand, their complements are eight pentatonic scales, whose harmonic characteristics are also analyzed and the corresponding parsimonious graph, called 5-Cyclops, is given. These graphs highlight the cy-cles of fifths and fourths, which are the only possible circumferences linking the same scale types in these graphs. Other parsimonious transformations, like moving one note by a whole tone, are easily found in these graphs, too. The acoustical relationship between those heptatonic and pentatonic scale types is analyzed by the pentachord-type vector, which lists the number of times each pentachord type is contained in a set type. With the inclusion of a musical ex-ample, all this information is intended mainly for theorists and composers.

Keywords*: Parsimonious Transformation, Heptatonic Scale, Pentatonic Scale, Cyclops, Trichord-Type Vector, Tetrachord-Type Vector, Pentachord-Type Vector, Cycle of Fifths, Cycle of Fourths*

4. ***An interactive tool for composing (with) automorphisms in the colored Cube Dance***

Alexandre Popoff, Corentin Guichaoua & Moreno Andreatta

The 'colored Cube Dance' is an extension of Douthett's and Steinbach's Cube Dance graph, related to a monoid of binary relations defined on the set of major, minor, and augmented triads. This contribution explores the automorphism group of this monoid action, as a way to transform chord progressions. We show that this automorphism group is of order 7776 and is isomorphic to $(\mathbb{Z}_3{}^4 \rtimes D_8) \rtimes (D_6 \times \mathbb{Z}_2)$ *. The size and complexity of this group makes it unwieldy: we therefore provide an interactive tool via a web interface based on common HTML/Javascript frameworks for students, musicians, and composers to explore these automorphisms, showing the potential of these technologies for math/music outreach activities.*

***Keywords**: Cube Dance · binary relations · monoid action · interactive software.*

5. ***Combinatorial Spaces***

Robert Peck

Combinatoriality—the property that obtains when unions of corresponding subsets within tone rows comprise aggregates—takes various forms, following the canonical operations that relate the constituent rows to one another: transposition, inversion, retrograde, and/or retro-grade inversion. The mathematical field of combinatorics presents tools to answer such basic questions as: How many combinatorial sets exist in a space of a given size? To how many equivalence classes do they belong? Such enumeration procedures involve various techniques that have prior connections to music theory. In the process of answering these questions, our results reveal further aspects of combinatorial sets. For instance, no combinatorial n-chords are held invariant by a translation operation with an odd index. The set of I-invariant n-chords that are P -combinatorial is equivalent to the set of those that are I-combinatorial, and this set is precisely the set of all-combinatorial n-chords. Such information sheds new light on these intriguing structures.

***Keywords**: Combinatoriality, serialism, combinatorics, enumeration*

6. ***Euler's "Tentamen"***

Sonia Cannas

The Tentamen novae theoriae musicae is a treatise in which Euler elaborated a new music theory using mathematics. The aim of this paper is to explain his theoretical system to justify the pleasure of listening to music and to analyze differences and similarities with other consonance theories.

***Keywords:** Euler, tentamen, consonance*

Categorical and Algebraic Approaches to Music

7. ***A Projection-Oriented Mathematical Model for Second-Species Counterpoint***

Octavio Alberto Agustín-Aquino & Guerino Mazzola

Drawing inspiration from both the classical Guerino Mazzola's symmetry-based model for first-species counterpoint (one note against one note) and Johann Joseph Fux's Gradus ad Parnassum, we propose an extension for second-species (two notes against one note).

***Keywords:** Second-species, counterpoint.*

8. When Virtual Reality helps fathom Mathemusical Hyperdimensional Models

Giles Baroin

Mathemusicians have always produced models for understanding, analyzing or computing music. We are used to visualize some of them on paper, in a theater or on a computer screen.

Even if they refer to multidimensional spaces (3D-4D), while displaying these models on a computer screen the viewer ends up with a 2D picture, or a movie. Planar projection limits the perception, nowadays, in the era of virtual reality, we propose tools and solutions to better apprehend these models and give the viewer an improved immersive experience.

Taking advantage of methods used in air traffic simulations, we are developing techniques that we will apply to existing mathemusical visualizations, beginning with Tonnetze and Hyperspheres.

We herewith introduce two recently revealed mathemusical models that we have created:

2D: The Shadow Tonnetz, our latest extension of the Tonnetz that keeps trace of a harmonic path.

4D: The Entangled Hyperspheres, a combination of two Planet-4D models that enables us to visualize microtonal music.

The images in this paper are extracted from immersive virtual reality world; during MCM we intend to presented the movies with adapted 3D equipment. All videos including virtual ones will be available on www.mathemusic.net.

***Keyword**s: Virtual Reality, Mathemusic, Visualization*

9. SUM Classes and Quotient Generalized Interval Systems

David Orvek & David Clampitt

The present paper develops algebraic properties of the SUM-class system first developed by Richard Cohn and explored by Robert Cook and Joseph Straus, in the context of David Lewin's Generalized Interval System (GIS) concept. Motivated by his observation that harmonic triads whose pitch classes sum to a given value modulo 12 share certain voice-leading properties, Cohn defined SUM classes for the 24 consonant (major and minor) triads, and defined transformations on these equivalence classes. We present the SUM-class system as a quotient GIS structure, and explore the dual quotient GIS implied by Lewin's theory for non-commutative GISs, and we generalize to other types of pitch-class sets (other set-classes).

***Keywords**: Generalized Interval System · group homomorphism · quotient group · SUM class*

10. Extended Vuza canons

Greta Lanzarotto & Ludovico Pernazza

Starting from well-known constructions of aperiodic tiling rhythmic canons by G. Haj\'os, N.G. de Bruijn and D.T. Vuza, several generalisations are given.

*In this way, it is possible to find new aperiodic canons, that we call **\emph**{extended Vuza canons}.*

***Keywords**:Mathematical models for music, Vuza canons, Aperiodic factorisations of cyclic groups.*

11. Some mathematical and computational relations between timbre and color

Maria Mannone & Juan Sebastián Arias-Valero

In physics, timbre is a complex phenomenon, as well as color. Musical timbres are given by the superposition of sinusoidal signals, corresponding to longitudinal acoustic waves. Colors are produced by the superposition of transverse electromagnetic waves in the domain of visible light. Regarding human perception, specific timbre variations provoke effects similar to color variations, for example, a rising tension or a relaxation effect. We aim to create a computational framework to modulate timbres and colors. To this end, we consider categorical groupoids, where colors (timbres) are objects and color variations (timbre variations) are morphisms, and functors between them induced by continuous maps. We also sketch some gestural variations of this scheme. Thus, we try to soften the differences and focus on the similarity of structures.

***Keywords**: color · timbre · topology · category theory · gestures*

12. Pairwise Well-Formed Modes as Transformations

Thomas Noll & David Clampitt

The paper extends the transformational approach to pair-wise well-formed (PWWF) modes, represented as words over a 3-letter alphabet. In particular it addresses open problems arising from an earlier approach (Noll, T and D. Clampitt. 2018. "Kaleidoscope substitutions and pairwise well-formed modes: Major-Minor duality transformationally revisited", Journal of Mathematics and Music. 12(3)), wherein kaleido-scope transformations were shown to generate PWWF modes, but it was not yet shown that all PWWF modes could be so generated. This gap is filled. A further problem is that the kaleidoscope transformations are not closed under composition. The final section introduces a new construction for the generation of PWWF words/modes, transformations of words on a 4-letter alphabet. A strongly-supported conjecture is that these transformations form a monoid.

***Keywords**: Pairwise Well-Formed Scales, Kaleidoscope Transformations, Diatonic and Syntonic Modes, Sturmian Morphisms, Algebraic Combinatorics on Words*

Applications of Mathematics to Musical Analysis

13. Mathematical Morphology Operators for Harmonic Analysis

Gonzalo Romero-García, Isabelle Bloch & Carlos Agon

Mathematical Morphology provides powerful tools for image processing, analysis and understanding. In this paper, we apply these tools to analyze scores, that are image-like representations of Music. To do that, we consider chroma rolls, a representation of scores similar to piano rolls that use chromas instead of pitches. Endowing this representation with a lattice structure, one can define Mathematical Morphology operators, and setting a group structure to the Time-Frequency plane allows us to use the notion of structuring element. We show throughout some examples that this relates with the notion of pitch-class set and chord progressions, and we analyze two Chopin's Nocturnes with this technique.

***Keywords**: Mathematical Morphology · Harmonic Analysis · Time-Frequency · Pitch-class Set · Chord Progression · Chroma Roll.*

14. Computational Analysis of Musical Structures based on Morphological Filters

Paul Lascabettes, Paul, Carlos Agon, Moreno Andreatta, and Isabelle Bloch

This paper deals with the computational analysis of musical structures by focusing on the use of morphological filters. We first propose to generalize the notion of melodic contour to a chord sequence with the chord contour, representing some formal intervallic relations between two given chords. By defining a semi-metric, we compute the self-distance matrix of a chord contour sequence. This method allows generating a self-distance matrix for symbolic music representations. Self-distance matrices are used in the analysis of musical structures because blocks around the diagonal provide structural information on a musical piece. The main contribution of this paper comes from the analysis of these matrices based on mathematical morphology. Morphological filters are used to homogenize and detect regions in the self-distance matrices. Specifically, the opening operation has been successfully applied to reveal the blocks around the diagonal because it removes small details such as high local values and reduces all blocks around the diagonal to a zero value. Moreover, by varying the size of the morphological filter, it is possible to detect musical structures at different scales. A large opening filter identifies the main global parts of the piece, while a smaller one finds shorter musical sections. We discuss some examples that demonstrate the usefulness of this approach to detect the structures of a musical piece and its novelty within the field of symbolic music information re-search.

Keywords: *Symbolic Music information research · Music structure · Chord contour · Self-distance matrix · Mathematical morphology.*

15. Non-Spectral Transposition-Invariant Information in Pitch-Class Sets and Distributions

Jason Yust & Emmanuel Amiot

The spectral information of a pitch-class set or distribution relates to its interval content and what Ian Quinn calls its harmonic qualities, the magnitudes of a discrete Fourier transform of a pitch-class vector. The spectrum is invariant with respect to transposition and in-version, but the existence of Z-related sets, which have equivalent spectra but are not related by transposition or inversion, means that the spectrum is not a complete description of a set class. We show how to isolate transposition-invariant phase information using products of Fourier coefficients. We describe some of the mathematical features of these coefficient products and show how they encode aspects of tonality, and can be useful for analyzing non-tonal music with an example from Takemitsu's "Air" for solo flute.

16. Tetrachordal Folding Operations

Jason Yust

Jonathan Bernard's trichordal folding operations relate tri-chords with a maximum of shared interval content. This paper generalizes this to any cardinality of chord, focusing on the case of tetrachordal folding. A tetrachordal folding holds one trichordal subset fixed and inverts another around a shared dyad, so that the two tetrachords share five interval classes and two trichordal subsets. These operations generalize naturally from pitch space to pitch-class space and to set classes. The last section of the paper demonstrates the analytical application of tetra-chordal folding networks on Morton Feldman's "For Stephan Wolpe."

***Keywords**: Pitch-class set theory · folding · interval content · Morton Feldman.*

Mathematical Techniques and Microtonality

17. Continuous Chromagrams and Pseudometric Spaces of Sound Spectra

Jordan Lenchitz & Anthony Coniglio

In this paper we extend the ubiquitous music information retrieval technology of the quantized chromagram (or chroma feature) to propose a continuous chromagram, an octave-reduced spectrogram that logarithmically reduces the frequencies of a sound spectrum onto a half-open interval [a, 2a) we call a chroma octave. We prove that for any real number r > 1, any two logarithmically reduced spectrograms onto intervals of reduction [a1, ra1) and [a2, ra2) with a1 ≠ a2 are equivalent up to logarithmic scaling and rotation. In the case r = 2 this proof shows why all chroma octaves bounded by both the upper and lower frequencies of the sound spectrum in question yield essentially the same continuous chromagram. We then propose a family of pseudometrics on sound spectra and discuss potential applications to analysis and composition.

***Keywords**: Continuous chromagram · Generalized octave reduction · Pseudometric spaces*

18. N2D3P9

Dave Keenan & Douglas Blumeyer

N2D3P9 is a mathematical function which was developed to help in designing the Sagittal microtonal music notation. Given a rational number n representing a pitch (relative to some tonic note), N2D3P9 estimates its rank in popularity among all rational pitches in musical use. A low value of N2D3P9 indicates that the ratio is used often, and so should have a simple accidental symbol, while a high value indicates that the ratio is used rarely and so can have a more complex symbol if necessary. It may also be useful in designing rational scales or tunings.

***Keywords**: microtonal · just intonation · Sagittal notation*

19. Performing Easley Blackwood's Twelve Microtonal Etudes: An Open-Source Software Development Project

Richard Leinecker & William Ayers

This paper outlines an open-source development project dedicated to the performance of microtonal music, specifically Easley Blackwood's Twelve Microtonal Etudes (1980). Despite the piece's fixed format, Blackwood stated a specific desire to have the work performed live. This project incorporates multiple elements to this effect, including a standalone software synthesizer and a web application. The paper details prior efforts for microtonal performance on traditional keyboard instruments by Joel Mandelbum and Robert Hasegawa, describes the emergent issues inherent in such endeavors to translate between standard performance practices and microtonal tunings, and proposes methods that will allow for the accurate and easily replicable performance of Blackwood's etudes. The software synthesizer, proposed as a pedagogical design project for computer science students at the University of Central Florida, is currently under active development. The web application, available at the project website, includes source code and documentation for the use of future development teams.

Keywords: *Microtonal, Equal temperament, Easley Blackwood, Performance, Keyboard, Gesture.*

Algorithms and Modeling for Music and Music-Related Phenomena

20. Spline Modeling of Audio Signals with Cycle Interpolation

Matthew Klassen

In this paper we introduce methods to model brief audio signals with cubic splines, presupposing a fundamental frequency f0. The signal is broken up into cycles using zero-crossings, and each cycle is modeled with a C2 cubic interpolating spline based on some target number of interpolation points. We reduce the data in the model by reducing the number of key cycles which are used to generate intermediate cycles by the method of interpolation of B-spline coefficients, or cycle interpolation.

Keywords*: Spline · Audio · Signal · Interpolation · Cycle*

21. Transposition and Time-Scaling Invariant Algorithm for Detecting Repeated Patterns in Polyphonic Music

Antti Laaksonen, Kiell Lemström & Otso Björklund

This paper presents an algorithm for the time-scaled re-peated pattern discovery problem in symbolic music. Given a set of n notes represented as geometric points, the algorithm reports all time-scaled repetitions in the point set. The idea of the algorithm is to use an onset-time-pair representation of music, which reduces the musical problem of finding repeated patterns to the geometric problem of detecting maximal point sets where all points are located on one line. The algorithm works in O(n4 log n) time, which is almost optimal because the size of the output can be Θ(n4). We also experiment with the algorithm using real musical data, which shows that when suitable heuristics are used to restrict the search, the algorithm works efficiently in practice and is able to find small sets of potentially interesting repeated patterns.

Keywords: *Music pattern discovery · Transposition and time-scaling invariance · Geometric algorithms.*

22. On the Memory Usage of the SIA Algorithm Family for Symbolic Music Pattern Discovery

Antti Laaksonen, Kiell Lemström

SIA is a fundamental algorithm in symbolic musical pattern discovery, which reports all maximal translatable patterns in a point set. The original SIA algorithm requires O(kn2 log n) time and O(kn2) space, where n is the number of points in the data set, and k is the number of coordinates in each point. In this paper, we present a sweepline algorithm that shares the running time of SIA but requires only O(kn) space, enabling to process of larger data sets without running out of memory. Since SIA is the first step in many pattern discovery tasks, our new algorithm can have a broad impact. For example, we discuss the problem of finding all occurrences of maximal translatable patterns with specific properties. We also compare the algorithms in practice and show that reduced memory usage can benefit real data sets.

Keywords*: Pattern discovery · SIA algorithms · Memory usage*

23. A proposal to compare the similarity between musical products. One more step for automated plagiarism detection?

Aarón López-García, Brian Martínez-Rodríguez & Vicente Liern

In previous works, the authors presented a measure of similarity between melodies by identifying them with sequences of ordered vectors and using a clustering process based on fuzzy logics. Along the same line, we propose a measure of musical similarity between fragments of digital audio. We present the SpectroMap algorithm that allows us to detect the local maxima of the audio spectrogram representation (also known as constellation map) and we compared the similarity between different maps belonging to different audio excerpts. As a result, it is obtained a value that represents the resemblance between two musical products. This procedure could be used as a non-subjective tool in automatic plagiarism detection. To illustrate this method, three experiments have been carried out comparing different versions famous pop songs. The results point to the usefulness of the method, although this should be contrasted with an analysis of the human perception of this similarity.

Keywords*: Fuzzy Clustering · Similarity · Plagiarism.*

24. A new fitness function for evolutionary music composition

<u>*Brian Martínez-Rodríguez*</u>

In this paper we propose a new ˝tness function for Evolutionary Computation purposes, based on a weighted by neighborhood average distance between two sequences of points within any metric space. We will apply this ˝tness function to the ˝eld of Computer-Assisted Composition focusing on the problem of thematic bridging, consisting in the evolutionary creation of a soft set of transitions between two given different melodies, the initial and the final one. Several self-adaptative strategies will be used to perform the search. A symbolic melody will be genotypically mapped into a sequence of genes, each of then containing the information of duration, frequency and time distance to following note. We will test the implementation of the fitness function by means of two experiments, showing some of the intermediate melodies generated in a successful run, and benchmarking every experiment with performance indicators for any of the three distinct evolutionary strategies implemented. The results prove this tness function to be a quick and suitable way for individual evaluation in genetic algorithms.

***Keywords**: Evolutionary Computation, Computer Assisted Composition · Fitness · Genetic Algorithm, Neighborhood, Thematic Bridging*

25. A mathematical model of tonal function: voice leadings

<u>*Francisco Gómez & Isaac del Pozo*</u>

This paper is the first in a series of two papers on mathematical models of tonal function. In this first paper, we present a mathematical model of tonal function whose scope is limited to classic music from the common practice period. After a formalization of some harmonic elements (pitch classes, chords, arrangements, voicings, voice leadings), the model of tonal function is described. Our model is based on voice leadings and the tonal function is defined in terms of them. A combinatorial optimization algorithm is used to determine the tonal function. In this work, only chords with the same number of voices are considered. The general case was left for the second paper of the series; in the second paper the model of tonal function is generalized.

***Keywords**: Tonal function, voice-leadings, chord progressions, matrix algebra, distance functions, nabla distance, optimal voice-leadings, chord classification, characteristic polynomials.*

26. A mathematical model of tonal function: modulation

<u>*Francisco Gómez & Isaac del Pozo*</u>

This work is the second part of a research into mathematical models of tonal function. In the first paper, we presented a mathematical model based on optimal voice leadings for chords with the same number of notes. We gave several examples for classical music in the common practice period. In this second paper, we generalize the model to jazz music and music from the extended common practice and we remove the constraint on the number of voices, allowing the classification of chords with a diﬀerent number of notes. Furthermore, modulation within this mathematical model is also explored and a classification of modulation between major and minor scales is constructed.

***Keywords**: Tonal function, voice-leadings, nabla distance, optimal* *voice-leadings, cadence endomorphisms, characteristic polynomials, modal in-terchange, modulation.*

27. ***Hypercube + Rubik's Cube + Music = HyperCubeHarmonic***

Maria Mannone, Takashi Yoshino, Pascal Chiu, Yoshifumi Kitamura

Musical chords and chord relations can be described through mathematics. Abstract permutations can be visualized through the Rubik's cube, born as a pedagogical device. Permutations of notes can also be heard through the CubeHarmonic, a novel musical instrument. Here, we summarize the basic ideas and the state of the art of the physi-cal implementation of CubeHarmonic, discussing its conceptual lifting up to the fourth dimension, with the HyperCubeHarmonic (HCH). We present the basics of the hypercube theory and of the 4-dimensional Ru-bik's cube, investigating its potential for musical applications. To gain intuition about HCH complexity, we present two practical implementations of HCH based on the tridimensional development of the hypercube. The first requires a laptop and no other devices; the second involves a physical Rubik's cube enhanced through augmented and virtual reality and a specifically-designed mobile app. HCH, as an augmented musical instrument, opens new scenarios for STEAM teaching and performing, allowing us to hear the "sound of multiple dimensions."

***Keywords**: Rubik's cube, hypergeometry, permutations, chords, ton-netz, mobile*

4.2 Abstracts for Posters

1. ***Persistent Homology on Musical Bars***
   Victoria Callet

   This poster presents a new way of building a filtered simplicial complex from a music piece and applying persistent homology in the context of musical analysis. Our approach consists of considering any musical score as the set of its musical bars, which we see as subsets of $\mathbb{R}^3$. With this definition, we may consider the Hausdorff distance between two musical bars, which gives us a point cloud from any score, and that allows us to build the associated Vietoris-Rips complex. We will then use barcodes to visualize persistent homology and give an illustration of our construction on a famous movie music piece.

   ***Keywords**: Musical bars, Filtered complex, Persistent homology, Bar codes, Musical analysis.*

2. ***Identifying Metric Types with Optimized DFT and Autocorrelation Models***
   Matthew Chiu & Jason Yust

This poster explores the classification of metric types using different feature representations. Using weighted timepoint, DFT, and autocorrelation, we train feedforward neural networks to distinguish allemandes, courantes, sarabandes, and gavottes in the Yale-Classical Archives Corpus. Autocorrelation and DFT models perform better than a baseline, with DFT consistently better by a small amount.

***Keywords**: Discrete Fourier transform · Autocorrelation · Meter classification · Metric types · Neural networks.*

3. ***Geometry of Music in the Parabola***
   Edgar Delgado

We develop a geometric analog of musical harmony from the group law of the affine parabola. First, we associate musical notes with points of parabola. Immediately, we can define the usual affine and linear transformations for musical chords in module theory. Subsequently, we show that the actions of the groups T /I in PK-nets, P LR, UT T s, and JQZ behave identically to the circumference space. Then, we propose to recreate the Planet-4D model, the study of musical distance and the DFT for subsets of points on the parabola. We believe that we have an innovative and motivational perspective to approach the traditional parabola in a musical meaning.

***Keywords:** Parabola · Group Law · Pitch-Class Set Theory · Affine Transformations · Neo-Riemannian Theory · Music Fourier Space*

4. ***midiVERTO: A Web Application to Visualize Tonality in Real Time ,***
<u>Daniel Harasim, Giovanne Affatato & Fabian Moss</u>

This paper presents a web application for visualizing the tonality of a piece of music—the organization of its chords and scales—at a high level of abstraction and with coordinated playback. The application applies the discrete Fourier transform to the pitch-class domain of a user-specified segmentation of a MIDI file and visualizes the Fourier coefficients' trajectories. Since the coefficients indicate different musical properties, such as triadicity and diatonicity, the application isolates as-pects of a piece's tonality and shows their development in time. The aim of the application is to bridge a gap between mathematical music theory, musicology, and the general public by making the discrete Fourier trans-form as applied to the pitch-class domain accessible without requiring advanced mathematical knowledge or programming skills up front.

Keywords: *Web application · Visualization · Discrete Fourier transform, Tonalidy, Midi*

5. ***Quantum-Musical Explorations on Zn***
<u>Thomas Noll</u>

Motivated through recent applications of quantum theory to the music-theoretical conceptualisation of tonal attraction, the paper recapitulates basic facts about quantum wave functions over the finite configuration space Zn, and proposes a particular musical application. After an introduction of position and momentum operators, the Fourier transform as well as the translation and undulation operators, particular attention is plaid to the Quantum Harmonic Oscillator via its Hamilton operator and its eigenstates. In this setup the time development of chosen wave functions is applied to the control of moving sound sources in a Spatialisation scenario.

Keywords*: Quantum Theory · Music Theory · Pitch Class Profiles.*

6. ***The Mystery of Anatol Vieru's periodic sequences unveiled***
<u>Luisa Fiorot, Alberto Tonolo & Riccardo Giblas</u>

Anatol Vieru proposed a compositional technique based on an algorithmic manipulation of periodic sequences in Z12. This technique was translated in mathematical terms. Two mathematical problems arose starting from the so called Vieru's sequence V : period of primitives and proliferation of values. In this poster we announce, providing only the sketch of the proofs, the solution of these questions in a purely algebraic way.

7. ***Benford's Law and Music***
Sybil Prince, Brian Wickman, Jack Null, Eric Gazin

We considered musical note frequencies of the 88 keys of the piano and found that they are Benford distributed. We extended our focus beyond the 88 keys and found the connection to Benford holds for the lowest note measuring at 16.35 Hertz (Hz) to the highest note of 7902.13 Hz. We next investigated whether the distribution holds within specific types of music. We found that classical music such as a random sample of songs from the Romantic period adhere to the Benford distribution while modern music such as a random sampling of songs from the 2000s do not. We also coined a term called "Naturalness" to assess how well a song adheres to the Benford distribution.

***Keywords**: Benford Distribution · logarithmic distribution of first dig-its · Classical Music.*

8. ***Altered Chord Alternatives***
Lauren Ruth

Motivated by the empirically pleasing sound of E♭7sus4 as a substitution for G7 in the II–V–I progression Dm7–G7–CM7, we ad-vocate for the use of this chord and four other uncommon four-note chord substitutions in jazz, taking Schönberg's schwebend as justification and explaining the details our MATLAB code for analyzing a four-note chord's "diatonic citizenship."

***Keywords**: Altered scale, Schönberg, schwebend, MATLAB, jazz.*

9. ***Information Synthesis of Time-Geometry QCurve for Music Retrieval***
Shanon Steinmetz & Ellen Gethner

We expand information segmentation to include additional properties of music geometry. We establish a distinct metric for invariant chord structure (harmonic consistency) and models for conjunct melodic motion and acoustic consonance. We combine these with centricity to form a unified measure of music geometry. Using geometric predictors and the LSQOP method, we classify music/non-music with comparable results to AI/ML, between 76% and 92% f-score.

***Keywords**: music geometry · information geometry · harmonic consis-tency · harmonic leading · centricity · dissonance · consonance · time-geometry · dissonance · qcurve · quant-curve · music retrieval · music detection · LSQOP.*

10. ***Investigating Style with Scale Embeddings***
Matt Chiu

In this poster, we use pitch-class vector embeddings to study scale relationships between composers. Recent research in natural language processing (NLP) has used machine learning to derive vector representations—known as embeddings—for words based on their co-occurrence. Borrowing from NLP, we use the word2vec algorithm to encode windows of pitch-classes, or pitch-class vectors, of music. We show that these embeddings not only replicate the well-known theoretical circle of fifths, but can also capture stylistic nuances between composers' use of scales.

***Keywords**: Scale theory · embeddings · word2vec · vector space · style*

4.3 Outreach Activities

Math+Music@MODA

- *Jeremy Kastine, "**Can You Canon?**"*

  An activity about composing canons with monophonic composite texture. Participants learn how this problem can be formulated in terms of finding maximal cliques of a graph.

- *Gilles Baroin, "**MatheMusical VR Movies and Interactive Models**"*

  A collection of mathemusical virtual reality movies. Participants use virtual reality headsets to clearly visualize concepts that would otherwise be difficult to explain and comprehend.

- *Paco Gómez, "**Matherhythm or rhythm is a killer**"*

  This activity puts forward mathematical content - exact division, division with remainder, greatest common divisor, Euclid's algorithm and evenness principle - along with musical content - time span, pulse, rhythm formation, and timelines -, and shows how those mathematical ideas can be used as a tool to understand music and also as a principle for composing music. Participants are able to perform music based on these concepts using Boomwhackers.

- *Maria Mannone, "**The CubeHarmonic**"*

  An activity about the "CubeHarmonic," a novel musical instrument employing the concept of the triad Tonnetz through the physical manipulations of the Rubik's Cube. Participants experience this instrument firsthand through two mobile apps developed by Maria's colleagues: Takashi Yoshino and Pascal Chiu.

- *Thomas Noll, "**The Collective Public Fourier Performance**"*

  In this activity, three participants control Fourier coefficients by holding flags at varying heights, which are interpreted by mobile devices and processed by a central computer, producing a histogram that indicates how loudly each of seven other participants are to play their assigned note of a diatonic scale

- *Luis Nuño, "**The Harmonic Wheel**"*

  An activity based on the "Harmonic Wheel," a physical tool that combines a Tonnetz transformed into a polar grid with a plastic disc containing the lines that define the major, harmonic and melodic minor scales, together with the scale degrees and the symbols of the corresponding seventh chords. The Harmonic Wheel is a powerful and versatile tool for analyzing and composing music, as well as providing an efficient mnemonic notation.