International Congress on Music and Mathematics

Puerto Vallarta, Mexico, November 26–29, 2014

Special theme
"Analogous Thought and Abstract Forms in Music"

Special panel
"Mathematics and Aesthetics in Julian Carrillo's (1875–1965) work"

General Document

Information, program, long-version abstracts, special panel.

(G. Pareyon, editor)

Scientific – Organizing Committee:

Octavio AGUSTÍN-AQUINO (Universidad de la Cañada, Oaxaca), Juan Sebastián LACH-LAU (Conservatorio de las Rosas, Morelia), Emilio LLUIS-PUEBLA (Faculty of Sciences, UNAM), Guerino MAZZOLA (University of Minnesota & Society for Mathematics and Computation in Music), Roberto MORALES-MANZANARES (Music Informatics Laboratory, LIM, at the University of Guanajuato), Pablo PADILLA-LONGORIA (Institute for Research on Applied Mathematics and Systems, IIMAS–UNAM), Gabriel PAREYON (CENIDIM–INBA, Mexico City).

Guerino MAZZOLA (Honorific president)
Emilio LLUIS-PUEBLA (Head of the Congress)
Gabriel PAREYON (Program chair)

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Guadalajara, Jalisco, México.

Contents of this program:

Welcome to the International Congress on Music and Mathematics at Puerto Vallarta!

It is my honor and pleasure to invite all scholars and artists who are engaged in the fascinating relation between mathematics and music to attend the International Congress on Music and Mathematics in Puerto Vallarta, Mexico, Nov. 26–29, 2014.

As we are moving into the 21st Century, studies on mathematics and music have established their presence in major annual conference venues, such as the International Computer Music Conference, the American Mathematical Society Conference, and the AMS/SEM/SMT Music Conference. In 2007 the Society for Mathematics and Computation in Music was founded, and related scholarly publications are documented in the *Journal of Mathematics and Music* and the *Computational Music Science* book series, together with the society's biannual conference since 2007.

The International Congress on Music and Mathematics in Puerto Vallarta in 2014 is a strong continuation of these events and a proof of the Mexican engagement in Mathematical Music Theory (MaMuTh), that started with MaMuTh talks and concerts at the 1st International Seminar on MaMuTh in 2000 in Mexico City and a Special Session at the Saltillo's Congress of the Mathematical Society of Mexico, and continued in the 4th International Seminar on MaMuTh in 2010 at Huatulco. This remarkable tradition, guaranteed by the congress head, mathematician at UNAM and classical concert pianist, Emilio Lluis-Puebla who organized the previous Mexican events, highlights the cultural counterpoint of scholarly endeavors and artistic creations.

Apart from well-known artistic interest in MaMuTh by famous composers, such as Iannis Xenakis, Pierre Boulez or György Ligeti, MaMuTh publications, projects, and creations testify serious mathematical culture, including combinatorics, the theories of words, graphs, groups, numbers, and module representations, algebraic and differential geometry, ODEs and PDEs, statistics, homological algebra, number theory, algebraic topology, category theory, and topos theory. This fact has attracted the interest of renowned mathematicians, such as Yves André, Pierre Cartier, Alain Connes, Alexander Grothendieck, Yves Helleguarch, and Yuri Manin.

MaMuTh is however not just applied mathematics, it is a mutual fertilization, similar to mathematics and physics, challenging maths in new concept architectures and models, such as gesture theory or Galois theory of concepts, and also regarding mathematical conjectures being enlightened by music-theoretical research. Moreover, musical creativity using software has strongly profited from mathematical modeling of analytical and compositional approaches for the working creative musician and music theorist.

In this sense, I wish you all the best in a shared effort and profit towards the 2014 venue in a spirit of unified human understanding.

Prof. Dr. Guerino Mazzola, School of Music, University of Minnesota.
Honorific president of the 2014 International Congress on Music and Mathematics.
President of the Society for Mathematics and Computation in Music.

Important dates calendar

2013		
June	21	First announcement. Poster & website launching.
2014		
March	02	Second announcement. General reminder & news publication.
March	23	Early registration opens (registration online).
June	22	Deadline for submission of individual paper proposals.
July	13	Deadline for submission of panel proposals and paper proposals within a panel.
July	18	Announcement of accepted proposals (of individual papers).
July	27	Announcement of accepted panel proposals and paper proposals within a panel.
August	18	Deadline for submission of individual papers (*program version*).
August	31	Announcement of accepted proposals of individual papers (*program version*).
September	08	Deadline for submission of papers within a panel (*program version*).
September	14	Announcement of accepted paper proposals within a panel (*program version*).
October	06	Deadline for submission of proposals for concerts and other cultural activities.
November	09	Announcement of accepted proposals for concerts and other cultural activities.
November	16	Final date for accommodation reservations.
November	19	Full program & abstracts released online.
November	22	Deadline for hotel payment.
November	26–29	**Congress development.**
December	3	Third announcement.
2015		
January	28	Deadline for submission of revised papers.
April	—	Proceedings edited by Springer, Berlin.

Schedules segmented by panels

November 26–29, 2014

Special theme:
"Analogous Thought and Abstract Forms in Music"

Special panel:
"Mathematics and Aesthetics in Julian Carrillo's (1875–1965) Work".

Plenary lectures (I – V)

Panels:

I. Dynamical Systems.

II. Logic, Algebra and Algorithmics.

III. Gestural Theories.

IV. New Methods for Music Analysis.

V. Modern Geometry and Topology.

				timeschedule
Wednesday 26	am	Arrival at the Hotel	Participants must register at the entrance of the Hilton's Convention Center, in front of the hotel's main entrance.	11:00 – 13:00
	am	Registration		12:00 – 12:45
	pm	**Opening ceremony** Room: Salon Vallarta C	Institutional welcoming.	13:00 – 13:15
	pm		**Gabriel Pareyon** (ICMM program chair) welcome lecture: "A Survey on the Mexican Tradition of Music and Mathematics".	13:15 – 13:30
	pm		**Emilio Lluis-Puebla** (ICMM head) welcome lecture: "On the Relationship Between Music and Mathematics".	13:30 – 13:45
	pm	**Plenary Lecture I** Room: Salon Vallarta C	**Guerino Mazzola** (ICMM Honorific president): "Gestural Dynamics in Modulation—A Musical String Theory".	14:00 – 15:00
	pm	Break	Lunch	15:00 – 16:30
	pm	SHUTTLE BUS to the University Campus	Transportation to the opening concert.	16:30 – 17:00
	pm	Concert Place: Auditorio Cifuentes Lemus, CUC – Universidad de Guadalajara	**Dr. Marco Antonio Cortés-Guardado** (Rector of the University of Guadalajara's Campus Puerto Vallarta) welcome speech. CONCERT: **Alacrán del Cántaro:** electroacoustic concert. **Guerino Mazzola** (piano): free jazz piano program.	17:00 – 19:30
	eve	SHUTTLE BUS to the Hilton Resort PV	Transportation back to the Hilton.	19:30 – 20:00
	eve	Toast and dinner (only for the Hilton's guests).	Puerto Vallarta's Hilton Resort.	20:00 – 21:00

Thursday 27	am	**Panel 1 on Dynamical Systems** Room: Salon Vallarta C	**Samuel Vriezen** (independent researcher, Amsterdam): "Diagrams, Games and Time". **Roberto Morales-Manzanares** (LIM – University of Guanajuato): "Compositional Generation of Macro-Structures with Dynamical Systems in my Opera *Dunaxhii*". **Elaine Chew** (Queen Mary University of London): "The Mechanics of Tipping Points: A Case of Extreme Elasticity in Expressive Timing". **Tom Johnson & Samuel Vrizen** (composers from Paris & Amsterdam): "Informal talk between Tom Johnson and Samuel Vriezen".	10:00 – 12:30
		Coffee break		12:30 – 13:00
	pm	**Plenary lecture II** Room: Salon Vallarta C	**Clarence Barlow** (University of California, Santa Barbara): "On the Structural and the Abstract in my Compositional Work".	13:00 – 14:00
		Break	Lunch	14:00 – 16:00
		Parallel sessions		16:00 – 18:00
		Panel 1 on Logic, Algebra and Algorithmics. Room: Salon Vallarta D	**Harald Fripertinger** (Karl-Franzens-Universität Graz) and **Peter Lackner** (Kunst-Universität Graz): "Tone Rows and Tropes". **Pauxy Gentil-Nunes** (Universidade Federal do Rio de Janeiro, UFRJ): "Partitiogram, Mnet, Vnet and Tnet: Embedded Abstractions Inside Compositional Games". **David Clampitt** (Ohio State University): "Lexicographic Orderings of Modes and Morphisms". **Daniel Moreira de Sousa** (Universidade Federal do Rio de Janeiro, UFRJ): "Textural Contour: A Proposal for Textural Hierarchy through the Ranking of Partitions Lexset".	16:00 – 18:00
		Panel 1 on Gestural Theories. Room: Salon Agave	**Carlos Mathias Motta** (Universidade Federal Fluminense, Niterói, Brazil): "Project DRUMMATH: Rhythms that Build Meaning in Mathematical Concepts for the Visually Impaired". **Yemile del Socorro Chávez-Martínez** (Faculty of Sciences, UNAM): "Mazzola's Escher Theorem". **Teresa Campos** (PMDM – UNAM): "A Proposal for Musical Writing for the Visually Impaired". **Juan Sebastián Arias** (Universidad Nacional de Colombia, Bogotá): "Gestures on Locales and Localic Topoi".	16:00 – 18:00

Thursday 27	pm	**Special panel (1): "Mathematics and Aesthetics in Julian Carrillo's (1875–1965) Work".** Room: Oficina	**Roman Brotbeck** (Bern University of the Arts, HKB, and Special panel's chair): "An Analytical-Comparative Approach to Carrillo's Metamorphosis and Wyschnegradsky's Non-Octaviant Spaces and their Reverberations". **Mario García Hurtado** (PMDM – UNAM): "Julián Carrillo's Numerical Notation in his Guitar Music: Challenges to the Interpreter and Performer". **Noah Jordan** (Composer and independent researcher, Vancouver, Canada): "Microtonality and the Music of Julian Carrillo from a Xenharmonic Perspective". **Francisco Guillermo Herrera-Armendia & Marcos Fajardo-Rendón** (Department of Mathematics, Escuela Normal Superior de México, Mexico City): "Digital Technology in Julian Carrillo's Microtonal Music Research".	16:00 – 18:00
	eve	Break		18:00
		Place: Rhythms Hilton's Lounge.	Colleagues informal meeting (only for the Hilton's guests).	18:15 – 19:30
		SHUTTLE BUS to the University Campus	Transportation to the concert.	19:30 – 20:00
		Concert Place: Auditorio Cifuentes Lemus, CUC – Universidad de Guadalajara	**Emilio Erandu** (piano): classical piano program. **Emilio Lluis-Puebla** (piano): program with Mexican historical repertoire.	20:00 – 22:00
		SHUTTLE BUS to the Hilton Resort PV	Transportation back to the Hilton.	22:00 – 22:30
Friday 28	am	**Plenary lecture III** Room: Salon Vallarta C	**Octavio Agustín-Aquino** (Universidad de la Cañada, Oaxaca): "Computational and Combinatorial Aspects of Counterpoint Theory".	09:40 – 10:40
		Plenary lecture IV Room: Salon Vallarta C	**Emmanuel Amiot** (IRCAM, Paris): "Viewing Diverse Musical Features in Fourier Space: A Survey".	10:45 – 11:45
		Coffee break		11:45 – 12:00
		Parallel sessions		12:00 – 14:00
		Panel on New Methods for Music Analysis Room: Salon Vallarta D	**Baqueiro-Victorin, Erik & Emil Awad** (Escuela Superior de Artes de Yucatán, Mérida, & Universidad Veracruzana, Xalapa): "Dramatic Time and Rhythmic Transformations on Elliott Carter's *Shard*". **Gareth Loy** (Gareth, Incorporated): "Music, Expectation, and Information Theory". **Mariana Montiel** (Georgia State University, Atlanta): "Manuel M. Ponce's piano *Sonata No. 2* (1916): An Analysis Using Signature Transformations". **Emilio Erandu** (Departamento de Matemáticas – CUCEI, Universidad de Guadalajara, Guadalajara): "A Group for Pitch Sequences Representation with Emphasis in Debussy's Music".	12:00 – 14:00

Friday 28	am	**Panel 2 on Dynamical Systems** Room: Agave	**Gérard Assayag** (IRCAM, CNRS, UPMC, Paris): "Creative Dynamics of Composed and Improvised Interaction". **Gabriel Pareyon** & **Silvia Pina-Romero** (Universidad de Guadalajara): "Phase Synchronization Analysis as Fingerprint Classifier for the Teponaztli's Timbral Features". **Mark Reybrouck** (University of Leuven): "The Musical Experience between Measurement and Computation: from Symbolic Description to Morphodynamical Approach". **Roberto Cabezas, Edmar Soria & Roberto Morales-Manzanares** (UNAM & Universidad de Guanajuato): "Dynamical Virtual Sounding Networks: An Algorithmic Compositional Structure Based on Graph Theory and Cellular Automata".	12:00 – 14:00
	pm	Break	Lunch	14:00 – 16:00
		Parallel sessions		16:00 – 18:00
		Panel 2 on Logic, Algebra and Algorithmics Room: Salon Vallarta D	**Miguel Angel Cruz-Pérez** (Escuela de Laudería – INBA, Querétaro): "Set Theory and its use for Logical Construction of Musical Scales". **Tsubasa Tanaka & Kouichi Fujii** (Tokyo University of the Arts & NTT DATA Mathematical Systems, Tokyo): "Melodic Pattern Segmentation of Polyphonic Music as a Set Partitioning Problem". **Jason Yust** (Boston University School of Music): "Restoring the Structural Status of Keys through DFT Phase Space". **Carlos de Lemos Almada** (Universidade Federal do Rio de Janeiro, UFRJ): "Gödel-vector and Gödel-address as Tools for Genealogical Determination of Genetically-Produced Musical Variants".	16:00 – 18:00
		Panel 2 on Gestural Theories Room: Agave	**Mauro Herrera-Machuca** (PMDM – UNAM): "Configuring the Mapping of Movement Attributes for the Transmission of *Meaning* Through Electroacoustic Music". **Jaime Lobato-Cardoso & Pablo Padilla-Longoria** (IIMAS – UNAM): "Models and Algorithms for Music Generated by Physiological Processes". **Martin Norgaard** (Georgia State University, Atlanta): "How Learned Patterns Allow Artist-Level Improvisers to Focus on Planning and Interaction During Improvisation". **Fernando Zalamea** (Universidad Nacional de Colombia): "Mazzola, Galois, Riemann, Peirce and Merleau-Ponty: A Triadic, Spatial Framework for Gesture Theory".	16:00 – 18:00
		Special panel (2): "Mathematics and Aesthetics in Julian Carrillo's (1875–1965) Work". Room: Oficina	**Juan Sebastian Lach Lau** (Conservatorio de las Rosas): "Compositional research into the logics of pitch-distance and the timbral facet of harmony in Julián Carrillo's Leyes de Metamorfosis Musicales (Laws of Musical Metamorphoses)". **Gabriel Pareyon** (CENIDIM): "Carrillo's vs. Novaro's Tuning Systems Nested within the Arnold Tongues". **Santiago Rovira-Plancarte** (Faculty of Sciences, UNAM): "Julián Carrillo's Microtonal Counterpoint".	16:00 – 17:30

Friday 28	eve	Break		18:00
		Place: Rhythms Hilton's Lounge.	Colleagues informal meeting (only for the Hilton's guests).	18:00 – 19:30
		SHUTTLE BUS to the University Campus	Transportation to the concert.	19:30 – 20:00
		Concert Place: Auditorio Cifuentes Lemus, CUC – Universidad de Guadalajara	**Mike Winter** (electronics):concert with recent compositions. **Samuel Vriezen** (piano):concert with Tom Johnson's *Chord Catalogue* (1986) and Vriezen's answer *Within Fourths/Within Fifths* (2006).	20:00 – 22:00
		SHUTTLE BUS to the Hilton Resort PV	Transportation back to the Hilton.	22:00 – 22:30
Saturday 29	am	**Plenary lecture V** Room: Salon Vallarta C	**Thomas Noll** (Escola Superior de Musica de Catalunya, Barcelona): "The Sense of *Subdominant*: A Fregean Perspective on Music-Theoretical Conceptualization".	10:00 – 11:00
		Coffee break		11:00 – 11:30
		Panel on Modern Geometry and Topology Room: Salon Vallarta C	**Micho Durdevich** (Institute of Mathematics, UNAM): "Music of Quantum Circles". **Jaime Lobato-Cardoso & Juan Antonio Martínez-Rojas** (ENM – UNAM & Departamento de Teoría de la Señal y Comunicaciones, Universidad de Alcalá, Madrid): "*Topos Echóchromas Hórou*: the Place of Timbre of Space". **Juan Sebastián Lach-Lau** (Conservatorio de las Rosas): "Proportion, Perception, Speculation: Relationship between Numbers and Music in the Construction of a Contemporary Pythagoreanism".	11:30 – 14:00
	pm	Break	Lunch	14:00 – 16:00
		Parallel sessions		16:00 – 18:00
		Panel 3 on Logic, Algebra and Algorithmics Room: Salon Vallarta D	**Franck Jedrzejewski** (CEA, Paris): "Algebraic Combinatorics on Modes". **Iván Paz** (Departament de Llenguatges i Sistemes Informatics, Technical University of Catalonia): "A Fuzzy Logic Approach of High Level Musical Features for Automated Composition Systems". **Julian Rohrhuber & Juan Sebastián Lach-Lau** (Institut Fuer Musik Und Medien, Robert Schumann Hochschule (Duesseldorf) & Conservatorio de las Rosas, Morelia): "Generic Additive Synthesis? Hints from the Early Foundational Crisis in Mathematics for Experiments in the Ontology of Sound". **Michael Winter** (University of Southern California & Wolfram Research, Champaign, IL): "On Minimal Change Codes for Generating Music".	16:00 – 18:00

Saturday 29	pm	**Panel 3 on Dynamical Systems** Room: Agave	**Mattia G. Bergomi** (IRCAM, Paris): "Dynamics in Modern Music Analysis". **Edmar Soria, Roberto Cabezas & Roberto Morales-Manzanares** (UNAM & Universidad de Guanajuato): "*Sonus Geometria*: A Theoretical Classification Model of Electroacoustic Concepts Based on Fundamentals of Topologic Dynamics". **E. Gerardo Mendizabal-Ruiz** (Departamento de Ciencias Computacionales – CUCEI, Universidad de Guadalajara): "A Computational Tool for Image Sound Synthesizing". **Goretti Paredes-Bárcenas & Jesús A. Torres** (Escuela de Laudería – INBA, Querétaro): "Comparison of Empirical and Specific Methods to Evaluate if the Construction of Free Plates of a Violin are Already Finished".	16:00 – 18:00
		Special panel (3): "Mathematics and Aesthetics in Julian Carrillo's (1875–1965) Work". Room: Oficina	**Roque Alarcón-Guerrero** (Faculty of Engineering, UNAM): "Mixed Mathematics: Documental Sources of Music, Medicine and Mathematics in Ignacio Bartolache's Works and Life". **Mariana Híjar-Guevara** (Facultad de Filosofía y Letras, UNAM): "Notes on the Aesthetic Dimensions of the Sound 13 Theory". **Lidia Ader** (Center for New Technology in the Arts "Art-parkING" & Nikolay Rimsky-Korsakov's Apartment and Museum, Saint Petersburg): "Sound Wars at the Turn of Epochs".	16:00 – 17:30
	eve	Break		18:00
		Place: Rhythms Hilton's Lounge.	Colleagues informal meeting (only for the Hilton's guests).	18:00 – 19:30
		Spare time		19:30 – 20:00
		CLOSING CEREMONY		20:00 – 22:00
		Place: Salon Vallarta C (at the Hilton's Convention Center)	**Noah Jordan** (guitar): program with experimental music. **Octavio Agustín Aquino, David López Caamal & Axel Álvarez** (guitars): Mexican and international repertoire. **Mario García Hurtado** (guitar): program with Julian Carrillo's music in quarter tones.	
		Formal closure	Words of the ICMM president and the Scientific – Organizing Committee representative	22:00 – 22:15
		Special guests dinner (only for the Hilton's guests; registration and pre-payment required for this event).		22:30 – 00:00
Sunday 30	am	**Departure**		early to 13:00

Lecturers by alphabetical order

(please note that lectures accepted for the special panel on Julian Carrillo and microtonality are listed on a separate table below this one).

Name	Institution	Title of the submitted paper
Agustín-Aquino, Octavio A.	Universidad de la Cañada, Oaxaca	"Computational and Combinatorial Aspects of Counterpoint Theory"
Almada, Carlos de Lemos	Universidade Federal do Rio de Janeiro (UFRJ)	"Gödel-vector and Gödel-address as Tools for Genealogical Determination of Genetically-Produced Musical Variants"
Amiot, Emmanuel	Institut de Recherche et Coordination Acoustique/Musique (IRCAM, Paris)	"Viewing diverse musical features in Fourier Space: a survey"
Arias, Juan Sebastián	Universidad Nacional de Colombia, Bogotá	"Gestures on Locales and Localic Topoi"
Assayag, Gérard	Institut de Recherche et Coordination Acoustique/Musique (IRCAM, Paris)	"Creative Dynamics of Composed and Improvised Interaction"
Baqueiro-Victorin, Erik & Emil Awad	Escuela Superior de Artes de Yucatán (Mérida, Yuc.) & Universidad Veracruzana (Xalapa, Ver.)	"Dramatic Time and Rhythmic Transformations on Elliott Carter's *Shard*"
Barlow, Clarence	University of California, Santa Barbara	"On the Structural and the Abstract in my Compositional Work"
Bergomi, Mattia G.	IRCAM, Paris	"Dynamics in Modern Music Analysis"
Cabezas, Roberto, Edmar Soria & Roberto Morales-Manzanares	UNAM & Universidad de Guanajuato	"Dynamical Virtual Sounding Networks: An Algorithmic Compositional Structure Based on Graph Theory and Cellular Automata"
Campos-Arcaraz, Teresa	PMDM – UNAM, Mexico City	"A Proposal for a Music Writing for the Visually Impaired"
Chávez-Martínez, Yemile del Socorro	Faculty of Sciences, UNAM (Mexico City)	"Mazzola's Escher Theorem"
Chew, Elaine	Queen Mary University of London	"The Mechanics of Tipping Points: A Case of Extreme Elasticity in Expressive Timing"
Clampitt, David	Ohio State University	"Lexicographic Orderings of Modes and Morphisms"
Cruz-Pérez, Miguel Angel	Escuela de Laudería – INBA, Querétaro	"Set Theory and its use for Logical Construction of Musical Scales"
Durdevich, Micho	Institute of Mathematics, UNAM, Mexico City	"Music of Quantum Circles"

Erandu, Emilio	CUCEI – Universidad de Guadalajara	"A Group for Pitch Sequences Representation with Emphasis in Debussy's Music"
Fripertinger, Harald & Peter Lackner	Karl-Franzens-Universität & Kunst-Universität, Graz	"Tone Rows and Tropes"
Gentil-Nunes, Pauxy	School of Music, Universidade Federal do Rio de Janeiro (UFRJ)	"Partitiogram, Mnet, Vnet and Tnet: Embedded Abstractions Inside Compositional Games"
Herrera-Machuca, Mauro	PMDM – UNAM, Mexico City	"Representing Body Gestures Through Sound: Strategies for Mapping Body Gestures Focusing on the Transmission of Meaning Through Electroacoustic Music Sounds"
Jedrzejewski, Franck	CEA, Paris	"Algebraic Combinatorics On Modes"
Johnson, Tom & Samuel Vrizen	Composers (Paris & Amsterdam)	"Informal talk between Tom Johnson and Samuel Vriezen"
Lach-Lau, Juan Sebastián	Conservatorio de las Rosas (Morelia)	"Proportion, Perception, Speculation: Relationship between Numbers and Music in the Construction of a Contemporary Pythagoreanism"
Lluis-Puebla, Emilio	Faculty of Sciences, UNAM, Mexico City	"On the Relationship Between Music and Mathematics"
Lobato-Cardoso, Jaime & Juan Antonio Martínez-Rojas	ENM – UNAM & Departamento de Teoría de la Señal y Comunicaciones, Universidad de Alcalá, Madrid	"*Topos Echóchromas Hórou*: the Place of Timbre of Space"
Lobato-Cardoso, Jaime & Pablo Padilla-Longoria	IIMAS – UNAM, Mexico City	"Models and Algorithms for Music Generated by Physiological Processes"
Loy, Gareth	Gareth, Incorporated (California)	"Music, Expectation, and Information Theory"
Mathias-Motta, Carlos	Universidade Federal Fluminense, Niterói (Brazil)	"Project DRUMMATH: Rhythms that Build Meaning in Mathematical Concepts for the Visually Impaired"
Mazzola, Guerino	University of Minnesota & ICMM 2014 Honorific president	"Gestural Dynamics in Modulation—A Musical String Theory"
Mendizabal-Ruiz, E. Gerardo	CUCEI, Universidad de Guadalajara	"A Computational Tool for Image Sound Synthesizing"
Montiel, Mariana	Georgia State University, Atlanta	"Manuel M. Ponce's piano *Sonata No. 2* (1916): An Analysis Using Signature Transformations"
Morales-Manzanares, Roberto	LIM – University of Guanajuato & PMDM – UNAM, Mexico City	"Compositional Generation of Macro-Structures with Dynamical Systems in my Opera *Dunaxhii*"

Moreira de Sousa, Daniel	Universidade Federal do Rio de Janeiro (UFRJ)	"Textural Contour: A Proposal for Textural Hierarchy through the Ranking of Partitions Lexset"
Noll, Thomas	ESMUC, Barcelona	"The Sense of *Subdominant*: A Fregean Perspective on Music-Theoretical Conceptualization"
Norgaard, Martin	Georgia State University, Atlanta	"How Learned Patterns Allow Artist-Level Improvisers to Focus on Planning and Interaction During Improvisation"
Paredes-Bárcenas, Goretti & Jesús A. Torres	Escuela de Laudería – INBA, Querétaro	"Comparison of Empirical and Specific Methods to Evaluate if the Construction of Free Plates of a Violin are Already Finished"
Pareyon, Gabriel	CENIDIM (Mexico City)	"A Survey on the Mexican Tradition of Music and Mathematics"
Paz, Iván	Departament de Llenguatges i Sistemes Informatics, Technical University of Catalonia	"A Fuzzy Logic Approach of High Level Musical Features for Automated Composition Systems"
Pina-Romero, Silvia & Gabriel Pareyon	CUCEI – Universidad de Guadalajara & CENIDIM (Mexico City)	"Phase Synchronization Analysis as Fingerprint Classifier for the Teponaztli's Timbral Features"
Reybrouck, Mark	University of Leuven (Belgium)	"The Musical Experience between Measurement and Computation: from Symbolic Description to Morphodynamical Approach"
Rohrhuber, Julian & Juan Sebastián Lach-Lau	Institut Fuer Musik Und Medien, Robert Schumann Hochschule (Duesseldorf) & Conservatorio de las Rosas (Morelia)	"Generic Additive Synthesis? Hints from the Early Foundational Crisis in Mathematics for Experiments in the Ontology of Sound."
Soria, Edmar, Roberto Cabezas & Roberto Morales-Manzanares	UNAM & Universidad de Guanajuato	"*Sonus Geometria*: A Theoretical Classification Model of Electroacoustic Concepts Based on Fundamentals of Topology Dynamics"
Tanaka, Tsubasa & Kouichi Fujii	Tokyo University of the Arts & NTT DATA Mathematical Systems, Tokyo	"Melodic Pattern Segmentation of Polyphonic Music as a Set Partitioning Problem"
Vriezen, Samuel	Independent researcher & composer, Amsterdam	"Diagrams, Games and Time"
Winter, Michael	University of Southern California & Wolfram Research (Champaign, IL)	"On Minimal Change Codes for Generating Music"
Yust, Jason	Boston University School of Music	"Restoring the Structural Status of Keys through DFT Phase Space"
Zalamea, Fernando	Universidad Nacional de Colombia, Bogotá	"Mazzola, Galois, Riemann, Peirce and Merleau-Ponty: A Triadic, Spatial Framework for Gesture Theory"

Lecturers for the special panel on Julian Carrillo:

Name	Institution	Title of the submitted paper
Ader, Lidia	Center for New Technology in the Arts "Art-parkING" & Nikolay Rimsky-Korsakov's Apartment and Museum, Saint Petersburg	"Sound Wars at the Turn of Epochs"
Alarcón-Guerrero, Roque*	Faculty of Engineering, UNAM (Mexico City)	"Mixed Mathematics: Documental Sources of Music, Medicine and Mathematics in Ignacio Bartolache's Works and Life"
Brotbeck, Roman	Bern University of the Arts, HKB (Switzerland)	"An Analytical-Comparative Approach to Carrillo's Metamorphosis and Wyschnegradsky's Non-Octaviant Spaces and their Reverberations"
García-Hurtado, Mario	PMDM – UNAM (Mexico City)	"Julián Carrillo's Numerical Notation in his Guitar Music: Challenges as an Interpreter and Performer"
Herrera-Armendia, Francisco Guillermo & Marcos Fajardo-Rendón	Escuela Normal Superior de México (Mexico City)	"Digital Technology in Julian Carrillo's Microtonal Music Research"
Híjar-Guevara, Mariana**	Facultad de Filosofía y Letras, UNAM (Mexico City)	"Notes on the Aesthetic Dimensions of the Sound 13 Theory"
Jordan, Noah	Composer and independent researcher, Vancouver, Canada.	"Microtonality and the Music of Julian Carrillo from a Xenharmonic Perspective"
Lach-Lau, Juan Sebastián	Conservatorio de las Rosas (Morelia)	"Compositional research into the logics of pitch-distance and the timbral facet of harmony in Julián Carrillo's *Leyes de Metamorfosis Musicales* (Laws of Musical Metamorphoses)"
Pareyon, Gabriel	CENIDIM (Mexico City)	"Carrillo's vs. Novaro's Tuning Systems Nested within the Arnold Tongues"
Rovira-Plancarte, Santiago	Faculty of Sciences, UNAM (Mexico City)	"Julián Carrillo's Microtonal Counterpoint"

* Lecture selected by its historical contents, connected as an antecedent for lecture**.

Abstracts by alphabetical order of authors:

Ader, Lidia
Center for New Technology in the Arts "Art-parkING"
and Nikolay Rimsky-Korsakov's Apartment and Museum, Saint Petersburg

Special panel "Mathematics and Aesthetics in Julian Carrillo's (1875-1965) work":

"Sound Wars at the Turn of Epochs"

Keywords: microtonal music, sound, atomic bomb, 20th century music, experimental music, *Zeitgeist*.

Abstract: The 20th century very beginning widely established a powerful symbol: the *atomic bomb*. Scientists studied its nature and development discovering contained energy; thereafter revolution went on very quickly. Physicists Ernest Rutherford and Frederick Soddy realized that, by breaking down, atoms could turn into new elements. These experiments completely changed people's minds and artists were the first to react. Positivist feelings in the creative environment of that time assigned a potential role to the natural-scientific method in the future of artistic development. Musicians, artists and poets addressed science in order to find an instrument of cognition or *common ground*. The *principle of division*, splitting a cellular whole into parts, attracted them. The concept of *dissociative identity disorder* and a the new term *Spaltung* unified a whole complex of notions: exfoliation, divarication, opposition and fissure. Such fissures marked many different spheres, particularly arts.

In my paper I will explain microtonality as a notion of division of a sound; a coherent item of separate fields—arts particularly. "Everything is analytically decomposing and dividing", stated philosopher Nikolay Berdyaev in his article *The crisis of the arts*. Simultaneously, Picasso proclaimed a "secret cosmic splitting" as an analytical decomposing process. Great thinkers thought that this would help artists to study the object as skeleton and as a solid, hidden matter behind its frame. Objective properties of things became a source for inspiration of artists. They operated primary elements of language; for example sounds, letters. The whole for them was just a secondary substance, derivative from the basis, *bones*. In other words, they proposed a new trend: dematerialization and subsequent reincarnation of the arts.

As far as scientific and artistic approaches were appearing swiftly and in remote spheres, it is hardly possible to think about succession, but rather of the phenomeon of *Zeitgeist*, i.e. *the genius of the time* as described by romantics. Composers in the beginning of 20th century proposed several sources for microtonal music: computation, imitation, addition, and enlargement. Here I will show how these methods works in music, with its powerful and weakest sides.

If we compare now some common processes in poetry and music of that time, we can consider that they had some corresponding methods: they saw signs of *restricting* consciousness in principles of organizing a whole. Word and sound were studied with the help of an auxiliary system, an artificially constructed structural model. An aim of such experiments was an attempt to search new criteria of art and to express it with the help of a language of mathematical formulae. They compared phoneme and sound with a physical agent. So, for example, they were understood as an *organic body*, or a *single organism*. Without this, a word would lack of individual qualities; moreover, a body is a primary cell developed upon any conditions; and then without a body a full-value existence and functioning of a word would be impossible, and if we consider its limitation by narrow borders, "the strength of it … is only mouldering" . In all manifestation of modernism, the notion of "grain" prevails. Artists developed a whole system of *agro-musical*, *-poetical*, *-art* equivalences. Sounds, phonemes and objects were all perceived as "grains of a language". The intention of their attempts was the creation of an algorithm of new structural construction; to plan a new 'periodic system' of word, sound, object.

Searching for destruction of integrity in the 20th century, one can divide this idea in two categories that differ by the type of material used. On the one hand, there are experiments, in which the division is a *system element*, base of work, method or principle of organization. This is in my opinion a basic function of division. On the other hand, divided elements are used for extremely accurate fixation of a text; they exist in art indirectly. They might be called as *application tools* of division.

Within basic principles there are several methods, used more often: coloristic and mathematical ones. Methods of development of material were formed during the long period of 12-tone temperament practice. However together with the introduction of microtones and new equal or not equal temperaments, there was necessity in modernization of traditional ways, taking into account new intonation possibilities. This is why there appear experiences of systems of comparison, melismatic development on the base of the smallest gradation of a tone, and study of sound spectrum. Using of mathematical techniques was due to striving for absolute accuracy and propriety of all experiments. Experiences in the field of temperament mostly focused around the 1920s.

As for the application of tools, we can look deeper into the experiments in the fields of acoustics and mathematics. Computation, calculation of tempered and non-tempered systems was temporary, however they gained success in 1920s mainly. Behind all works there was a desire for total accuracy and reasonability of all searches. While linguists discussed eternity of sounds in language, composers and scientists tried to build their own alphabet and to solve *a problem of sound division* in music.

However microtonal music was never accepted as a world trend. It always remained marginal, isolated. Even those composers who composed most of their works using microtones, usually marked those pieces by some constant *labels*: "piece for the violin and quarter-tone piano"; "…for 31-tone ensemble", etc. This was done facing performers and listeners, to gain attention and to reject those who do not accept it.

All this is also noticeable, not only by the distinction in titles regarding the *atomic era*, but by the musicians' perception: in Georgy Rimsky-Korsakov, for example, when calling "simple" those pieces written in 12-tone system. Another *classification* was given by the director of Universal Edition, Emil Hertzka, in a letter to Donaueschinger Musiktage director Mr. Burkard: "*Haba's II. Quartett im Viertelton-System ist allerdings noch nicht erschienen. <…> In wenigen Tagen dürfte Haba's I. Quartett im Normal ton-System erscheinen und ich werde mir erlauben, Ihnen eine Partitur zur Verfügung zu stellen*". For criticists it was clear that "it is still too early" to predict how this cult will develop. Even though in the beginning of 20th century, the *trend for development* was accompanied by violent, brutal attacks. The main criticisms were due to the restrictive framework of tonality, and the fear of developing music using quartertones. However sound wars did not pass unnoticed. Nowadays it is time to trace its impact in the development of music and art in general, to see its path in history conditioning the *atoms of future*.

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PLENARY LECTURE:

"Computational and Combinatorial Aspects of Counterpoint Theory"

Keywords: counterpoint, strong dichotomy class.

Abstract: The study of *strong dichotomy classes* (an abstraction for consonances and dissonances) lead to problems in additive combinatorics and algorithmic searches whose solution allows us to construct extensions that preserve the contrapuntal structure from one space into another, such that their outcome have desirable properties in the injective limit. We will discuss some recent advances on these matters, open problems and some consequences for certain issues in counterpoint theory in particular, and for mathematical musicology in general.

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Special panel "Mathematics and Aesthetics in Julian Carrillo's (1875-1965) work":

"Mixed Mathematics: Documental Sources of Music, Medicine and Mathematics in Ignacio Bartolache's Works and Life"

Keywords: Bartolache, music, mathematics, history, Enlightenment.

Abstract: This paper focuses on the life and works of José Ignacio Bartolache, a seminal figure of New Spain's enlightenment age. He published *Mathematical lessons,* the first of its type, in 1769, and *Flying Mercury*, a magazine that covered issues about chemistry, medicine and physics. He was completely devoted to the spreading of scientific knowledge and made of mathematics the cornerstone for the development of the other sciences. In *Mathematical lessons* he called Music and Acoustics as "mixed mathematics" by their dependence to the principles and rules of thinking of the mathematical methods.

This paper makes hypothetical connections between Bartolache's life and works with the ideas displayed in the books listed in the inventory of his goods. As a doctor he believed that music could serve as a remedy against public diseases. In *Flying Mercury* he used musical terms as metaphors for the description of scientific instruments. In *Mathematical Lessons* he stated that music was formed by pleasant sounds and as a musician maybe he enjoyed the mixture of music theory (counterpoint and harmony), medicine (theory of human humours) and mathematics (logarithms) that one can find in Pablo Nasarre's *Musical fragments* (1693) and *Musical school according the modern practice* (1724) along with Rameau's *Treatise of harmony* (1722) and *Universal music or universal principles of music* (1717) by Pedro de Ulloa.

A learned and practical musician in his personal library there were musical scores composed by Giussepe Valentini, Leonardo Leo, an opera by Corelli, a flute sonata by Locatelli and a sketch of Ignacio Jerusalem's *First Lamentation for Holy Wednesday* that reveals us the influence of the Italian school of the 18th century in the musical taste of the New Spain. He knew how to play the guitar thanks to Santiago de Murcia's *An explanation for playing the part with the guitar* (1714) and the recorder in Cayetano Echeverría's *A book with lessons and sonecitos for recorder.* At his home there were a cello with its bow, a transverse flute, a theorbo, an 'orchestra' (a mechanical music device) and a bandora. Bartolache's spiritual life had a musical mirror in the *Art of plainsong* (1705) by Francisco Montanos, Joseph de Torres' *The practical art of organ's singing* (1705), and a *Musical catechism* by José Antonio Onofre de la Cadena (1763).

This research is supported by musical iconographic sources and its aim is set a background for Bartolache's works. Living in a time of shifting paradigms, José Ignacio Bartolache was a man between the worlds of faith against science, and in the crossroads of art, mathematics and music regarded both as entertainment and a source of knowledge.

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"Gödel-vector and Gödel-address as Tools for Genealogical Determination of Genetically-Produced Musical Variants"

Keywords: Gödel-vector, Gödel-address, genetic Algorithms, developing variation, *Grundgestalt.*

Abstract: The present paper integrates a broad research project which aims at a systematical production of musical variants through employment of genetic algorithms. The structure of this system is theoretically grounded on the principles of developing variation and *Grundgestalt*, originally elaborated by Arnold Schoenberg. Both principles, in turn, derive from a conception based on the trend of Organicism, which exerted a strong influence on the musical creation of Romantic Austro-German composers in 19th Century (MEYER, 1989, p.190). According to this conception, a musical piece must be created as an organic form, like a tree from a seed (which represents approximately the concept of *Grundgestalt*), through an continuous growing process, based essentially on sequential and/or recursive application of variation techniques, which can yield, at least in an idealized case, all the needed melodic material for the piece. Such a maximally economic procedure corresponds

essentially to the developing variation principle.[1] Based on a conceptual and terminological *corpus* derived from a former model elaborated for analytical finalities in the first phase of the research,[2] a new approach was established, specifically dedicated to the organic composition. For this purpose, it was created a genetic algorithm complex (the geneMus complex, or gM) formed by four computational complementary and sequential modules, which are destined to the systematical production of lineages of variants.[3] In the first module of gM, a brief monophonic musical fragment – the *Grundgestalt*, or, in the research's terminology, the *axiom* of the system – becomes a referential form for the production of a first generation of abstract variants (i.e. rhythmic and intervallic separate transformations, named as *geno-theorems*, or gT's) by application of some *rules of production* (the correspondent term in the system for the developing variation techniques). The first generation's gT's, in turn, become referential forms for the production of a second generation of gT's, by recursive and/or sequential application of the same precedent rules of production. The process can then be indefinitely replicated, resulting in extensive lineages of abstract derived forms. In the second module, pairs of gT's are crossed over, forming concrete musical unities, named as *pheno-theorems* (pT's). The third module is responsible for the concatenation of two or more pT's into large and more complex musical structures, labeled as *axiomatic groups* (axG's). Each axG produced can be considered as a "patriarch" of a specific lineage of variants (*group-theorems*, or thG's), which are produced in the fourth module, by sequential and/or recursive application of new rules of production (including some "mutational" ones, i.e., affecting only random selected elements), through at most seven generations (this number was arbitrarily chosen). An indispensable need that arose in the research was a precise mean for classifying the thG's produced in the system in such a way that their respective "genealogical" position and derivative order could be adequately preserved and retrieved when desired. For this purpose, it was created the *Gödel-vector* (Gv), with seven entries, each one representing one of the possible generations for the *groups* (i.e., a generic category that encompasses both axG's or thG's). The sequence of integers in the seven Gv's entries of a given group represent not only its own order of appearance, but also those of all its predecessors (zeros indicate none descents). Be, for example, the following groups and their respective Gv's: (a) <**1 0 0 0 0 0 0**> and (b) <**2 1 3 1 4 1 0**>. Gv (a) represents the first produced axG (or else, the patriarch of lineage 1), since the zeros indicate it has no descents in the subsequent six generations. Gv (b) corresponds to a thG of fifth generation. Its genealogy description is somewhat complex (it must be read from right to left): first descent (of generation 5) of the fourth descent (of generation 4) of the first descent (of generation 3) of the third descent (of generation 2) of the first descent (of generation 1) of the second axG. A special algorithm was designed to translate a Gv into a univocal index – the *Gödel-address* (Ga) – which represents concisely, as a unique integer, the precise genealogical identification of the considered group. It is calculated in the following manner: the numbers present in the seven entries of a Gv become exponents of the ordered seven first prime numbers (2, 3, 5, 7, 11, 13, 17). The product of these factors is equal to the correspondent Ga. This procedure is based on that one elaborated by the Austrian mathematician Kurt Gödel (whose name is used for label both concepts, Gv and Ga) for obtaining the *Gödel Number* corresponding to a given logical proposition in the Number Theory (NAGEL & NEWMAN, 2001). Considering the two Gv's above exemplified, the calculation of their respective Ga's proceeds as follow:

(a) <**1 0 0 0 0 0 0**> → $Ga_a = 2^1.3^0.5^0.7^0.11^0.13^0.17^0 = 2$;

(b) <**2 1 3 1 4 1 0**> → $Ga_b = 2^2.3^1.5^3.7^1.11^4.13^1.17^0 = 1{,}998{,}496{,}500$.

Therefore, there exists a proportional relationship between the genealogical complexity of a given group and the size of its respective Ga. The inverse process (i.e., the retrieval of a Gv from a given Ga) is easily realized with a simple factoring algorithm: the exponents of the ordered prime factors obtained become precisely the vector content. Another algorithm is employed for classifying the thG's according to their genealogy and to subordinate them to their respective axG's, thus constituting a a large matrix named *LEXICON*.

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[1] For more detailed information about the principles of *Grundgestalt* and developing variation, see, among others, Carpenter (1983), Frisch (1984), Dahlhaus (1990), and Dudeque (2005).

[2] For some published papers with analytical model studies, see Almada (2011a; 2011b; 2013c).

[3] For some published papers concerning this compositional approach, see Almada (2012; 2013a; 2013b).

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PLENARY LECTURE:

"Viewing Diverse Musical Features in Fourier Space: A Survey"

Keywords: Fourier, DFT, musical scales, rhythmic tilings, even distribution, homometry.

Abstract: In the last decade, Discrete Fourier Transform (or DFT) of musical structures has come to the fore in several domains and appears to be one the most promising tools available to researchers in music theory.

From David Lewin's very first paper (1959) and its revival by Ian Quinn (2005) it came to be known that the magnitude of Fourier coefficients tells much about the shape of musical structure, be it a scale, chord, or (periodic) rhythm: distributions with equal magnitude of Fourier coefficients are *homometric*, a (slight) generalization of isometric which was first studied in crystallography.

Maximality of some Fourier coefficients (saliency) characterizes very special scales, such as maximally even distributions. On the other hand, flat distributions of these magnitudes can be shown to correspond with uniform intervalic distributions. This type of analysis can be extended to labelled collections, enabling for instance comparisons of tunings. Furthermore, nil Fourier coefficients are highly organized and play a vital role in the theory of tilings of the line, aka rhythmic canons.

Finally the cutting edge research on the other component of Fourier coefficients, their phases, appear to model some aspects of tonal music with unforeseen accuracy. Some or all of these aspects can be extended from the discrete to the continuous domain.

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"Gestures on Locales and Localic Topoi"

Keywords: Gestures, locales, localic topoi, Grothendieck topos, diamond conjecture.

Abstract: The theory of gestures has meant a revolution for the mathematical music theory established by Guerino Mazzola in his famous book *Topos of Music* [1] in 2002. In several publications, he has presented a solid framework for the definition of mathematical gesture from three points of view: music, philosophy and mathematics. This definition is formulated originally for topological spaces and topological categories [3]. The iteration of gestures leads to the construction of hypergestures [3], using tools from classical homotopy theory.

In this article we expose a generalization of mathematical gestures on topological spaces introduced by Mazzola in [2], to locales and categories of sheaves on locales. In first place, we consider a recapitulation of Mazzola's construction in terms of

exponentials and limits in the category of topological spaces. Secondly, we show how are possible these constructions in the category of locales and the category of localic topoi (categories of sheaves on locales), and then we present examples of mathematical and musical interest. The constructions of exponentials of locales are based on an article of Hyland [4], and Johnstone's *Sketches of an Elephant* [5]. The constructions of limits of locales are taken from the book *Categorical Foundations* [6]. The respective constructions in the category of localic topoi are a consequence of the equivalence of this category and the category of locales [5]. We want to stress that our general construction of gestures takes into account that of hypergestures.

Finally, we comment the possible generalizations to sites, Grothendieck topoi and elementary topoi, which are our ongoing work. In this way will be discussed subsequent implications on Mazzola's architecture of mathematical music theory based on the topos structure [1], and the diamond conjecture stated on [2]. The emergence of Grothendieck topoi and elementary topoi in gesture theory could help to recast the diamond conjecture in an abstract setting, perhaps easier to handle.

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"Creative Dynamics of Composed and Improvised Interaction"

Keywords: digital intelligence, artificial creativity, creative agents, improvisation, musical formal structures.

Abstract: Until recently, in the field of musical interaction with machines, engineers and researchers have been concerned by fast computer computation and reaction —a logical concern considering available machine speeds and complexity of tasks. However, instantaneous response is not always the way a musician reacts in a real performance situation. Although decisions are being carried out at a precise time, the decision process relies on evaluation of past history, analysis of incoming events and anticipation strategies. Therefore, not only can it take some time to come to a decision, but part of this decision can also be to postpone action to a later time. This process involves time and memory at different scales, just as music composition does, and cannot be fully apprehended just by conventional signal and event processing.

In order to foster realistic and artistically interesting behaviors of digital interactive systems, and communicate with them in a humanized way, we wish to combine several means: machine listening —extracting high level features from the signal and turning them into significant symbolic units; machine learning —discovering and assimilating on the fly intelligent schemes by listening to actual performers; stylistic simulation —elaborating a consistent model of style through mathematical formalization; symbolic music representation —as formalized representations connecting to organized musical thinking, analysis and composition. These tools cooperate in effect to define a multi-level memory model underlying a discovery and learning process that contributes to the emergence of a creative musical agent.

In the Music Representation Team, after OpenMusic, a standard for computer assisted composition and mathemusical tools, we have designed OMax, an interactive machine improvisation environment which explores this new interaction schemes. It creates a cooperation between heterogeneous components specialized in real-time audio signal processing, high level music representations and formal knowledge structures. This environment learns and plays on the fly in live setups and is used in many artistic and musical performances.

Starting from OMax, we show recent trends of our research on interactive creative agents capable of adequacy and relevance by connecting instant contextual listening to corpus based knowledge, with longer term investigation and decision processes allowing to refer to larger-scale structures and scenarios. We call this scheme Symbolic Interaction. Creative Symbolic Interaction brings together the advantages one can get from the worlds of interactive real-time computing (the mathematics of signal) and intelligent, content-level analysis and processing (the mathematics of symbolic forms), in order to enhance and humanize man-machine communication. Performers improvising along with Symbolic Interaction systems experiment a unique artistic situation where they interact with a musical (and possibly visual) agent which develops itself in its own ways while keeping in style with the user. It aims at defining a new artificial creativity paradigm in computer music, and extends to

other fields as well : The idea to bring together composition and improvisation through modeling cognitive structures and processes is a general idea that makes sense in many artistic and non-artistic domains. It is a decision-making paradigm where a strategy makes its way weaving decisions step after step, either by relating to an overall structural determinism, or by jumping in an "improvised" way and generate a surprise.

This kind of "improvisation" strategy is observed in the living world, and might be one aspect of intelligence as a way to cope effectively with the unnown, it may serve as a productive model for artificial music creativity.

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"Dramatic Time and Rhythmic Transformations on Elliott Carter's Shard"

Keywords: Elliott Carter, transformational theory, tempo modulation, musical form, time-point classes.

Abstract: Since the second half of the 20th century, the concept of *musical time* represents a fundamental research theme on music theory. The main goal of this paper is to formally analyze *Shard* (1997) for solo guitar, by Elliott Carter (1908–2012), with special focus on time domain.

With the aim of generating a theory of musical form on Carter's repertoire, we built modulation and phase-shift networks among certain *tempi* which evinces rhythmic relationships at different structural levels. Those transformations are underpinned on a pair of Generalized Intervals Systems (GIS) generated in this analytical process.

1. Rhythmic Transformational Theory

Definition 1. Let be a any time-point (a real number) and d its duration (a positive real number). We define the a-class module d as the family of all time-points which are situated from the reference point a in a distance of a multiple of d, i.e.,

$$[a]d = \{x : x - a = nd, \text{ for some integer } n\}. \quad (1)$$

Definition 2. We define the *tempo* generated by the time-span (a, d) as the family of each time-span $(x, y) \in$ TMSPS whose first component (the time-point x) belongs to $[a]_d$,

$$\tau\,(a, d) = \{(x, y) : | x\, [a]_d\}. \quad (2)$$

Definition 3. Let be $\mathscr{T}$ the family of all *tempi* characterized as definition 2. We define a transformation

$$i_M : \mathscr{T} \times \mathscr{T} \to \mathbb{R}^+$$

as

$$i_M\,(\tau(a, x), \tau(b, y)) = \frac{x}{y}, \quad (3)$$

for any $\tau(a, x)$ and $\tau(b, y)$ in $\mathscr{T}$. In this case we say that $\tau(a, x)$ is transformed to $\tau(b, y)$ through the modulation $\frac{x}{y}$.

Theorem 1. *$(\mathscr{T}, (\mathbb{R}^+, \cdot), i_M)$ is a GIS.*

Definition 4. Let be $d \in \mathbb{R}^+$ a fixed duration and $\mathscr{T}(d)$ the family of all *tempi* whose generators posses d as its duration. We define a function

$$i_P : \mathscr{T}(d) \times \mathscr{T}(d) \to \mathbb{R}_d$$

as

$$i_P(\tau(a, d), \tau(b, d)) = [b - a]_d \tag{4}$$

for any $\tau(a, d)$ and $\tau(b, d)$ in $\mathscr{T}(d)$. In this case, we say that $\tau(a, d)$ is transformed to $\tau(b, d)$ through the phase-shift $[b - a]_d$.

Theorem 2. *For every duration* $d \in \mathbb{R}^+$, $(\mathscr{T}(d), \mathbb{R}_d, i_P)$ *is a GIS.*

2. Tempi Transformational Networks

Table 2 evinces how d_0 and d_1 are graphically represented through bars 1 to 15 whereas figure 1 depicts the two kinds of *tempi* transformations above mentioned from $\tau(0,4d_0)$ to $\tau(d_0,4d_0)$ at different levels:

Modulation: $\tau(0,4d_0)$ is transformed to $\tau(Y_1(D), 5d_1)$ through a $^8/_{15}$ interval, i.e., a local *rallentando* occurs at the presentation of the all-interval tetrachord $Y_1(D) = \{D, E\flat, F, A\}$ on bar 8. The corresponding inverse transformation ($^{15}/_8$) occurs from bar 8 to 15 returning to the original tempo in terms of velocity.

Phase-shift: $\tau(0,4d_0)$ is transformed to $\tau(d_0,4d_0)$ through a $[d_0]_{4d0}$ displacement (notated simply as d_0), i.e., a local change of the *thesis* (strong beat) is presented in this passage which connects the all-trichord hexachords $H(D)$ and $H(E\flat)$.

Would the original *thesis* be perceived again? How do the different *tempi* modulations contribute to this hypothetical objective?

Dramatic Time and Rhythm Transformations on Elliott Carter's *Shard* :

Table 1 Representations of d_0, d_0.

bars	MM	d_0	d_1
1-4	♩=108	𝅘𝅥𝅯	
4-14	♩=144	𝅘𝅥𝅮 (3)	𝅘𝅥𝅮
15-17	♩=108	𝅘𝅥𝅯	

$$3d_0 = 2d_1 \tag{5}$$

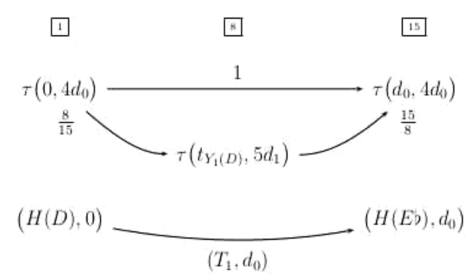

Fig. 1 Rhythmic transformations on *Shard*, bars 1–15.

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Barlow, Clarence
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PLENARY LECTURE:

"On the Structural and the Abstract in my Compositional Work"

Abstract: From 1959 to 1969 I composed music as most others do and have done – by direct transferenence from the imagination to a musical instrument (in my case the piano) and from there to a written score. During this period, I found myself relying increasingly on traditionally structured techniques such as canon, fugue, dodecaphony, serialism and electronics. In 1970 I was struck for the first time by a mathematical rule-based idea for an ensemble piece, which necessitated my learning to program a computer. Since then I have composed over fifty works (half my total output) with computer help – works for piano, organ, chamber ensemble, orchestra and electronics. Of these fifty-odd pieces, nine are partially and sometimes wholly based on abstract, exclusively mathematical principles:

Cheltrovype (1968-71) for cello, trombone, vibraphone and percussion, in a part of which the melody instruments each follow a probabilistic pitch distribution system based on an exponential curve, a sine curve and a parabola,

Sinophony II (1969-72) for eight-channel electronics, in which theoretically infinitely many but factually about 800 sine tones move along predetermined sinusoidal paths in the domains of pitch, amplitude and duration,

Stochroma (1972) for solo piano, a conceptual piano piece in which pitch, loudness and duration are probabilistically determined, allowing duration and dynamic values to exponentially diverge (as powers of 0.5) from a central value to rare but great extremes (durations for instance range in seconds from the yocto to the yotta range and beyond in both directions),

Bachanal for Jim Tenney and Tom Johnson (1990) for solo piano based on the digits of the natural number series in binary form,

Piano Concerto #2 (1961-1998) for piano and orchestra, in a part of which different groups of instruments perform simultaneous but independent accelerandi and decelerandi deriving from the shape of an inverted cosine,

Les Ciseaux de Tom Johnson (1998) for chamber ensemble based on the successive positions of six sets of three points derived from the name of the dedicatee, each set moving along a differently sized circle,

"...or a cherish'd bard..." (1999) for solo piano based on the rising gradient of repeated, theoretically infinitely long note chains derived in pitch from the spelling and in rhythm from the hexadecimal interpretation of the dedicatee's name,

Approximating Pi (2007) for up to 16 channels of electronics based on the numerical digits of convergences to the constant π as generated by the Madhava-Leibniz series,

Songbird's Hour (2011) for one channel of electronic sound based on my own music composition system of mathematically interpolating phase-contigual sine curve segments between the samples of a digitized sound wave in order to interpret the sound wave as a pitch sequence transposable into the audible range.

This talk will describe these nine pieces or relevant sections of them in varying detail.

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"Dynamics in Music"

Abstract: In the first part of this work we will study a geometrization of the Tonnetz. The space is naturally isotrpopic. Dissonance gives a method to introduce preferred directions in a simplicial representation on the Tonnetz. In the second part, starting from continuous musical models, we introduce a braids-theoretical interpretation of voice leading theory, focusing our attention on voice-crossing analysis.

Discrete Models: the Tonnetz and the dissonance

The Oettingen Riemann Tonnetz [Euler, 1739], i.e. the Tonnetz whose directions are minor third, major third and perfect fourth intervals, denoted $T(3,4,5)$ is often described as a torus. We can always unglue the torus in a triangular lattice, and find a minimal set of vertices such that the whole chromatic scale $\mathscr{C}$ is represented. See figure 1. This kind of representation can be used on the standard generalization of the Tonnetz given by:

$$\mathscr{T} = \left\{ T(i_1, i_2, i_3) : \sum_{k=1}^{3} i_k = 12 \right\}. \tag{1}$$

In this generalized context it is not always possible to find the whole chromatic scale in the set of vertices of $\mathscr{T}$. It suffices that i_k is not a generator of $\mathbb{Z}/12\mathbb{Z}$ for $k \in \{1,2,3\}$. Even in these cases it is possible to find $S \subset \mathscr{C}$ which are musically relevant: in Tonnetz like $T(2,2,8)$ one can find the *whole tone scale*. [See: Bigo *et al.*, 2013 for further details].

The idea is to consider the Tonnetz as a 2-dimensional simplicial complex embedded in $\mathbb{R}^2$ whose 2-simplices are the triangles (representing major and minor triads in the case of $T(3,4,5)$) and whose vertices are labeled with the name of the notes p they represented in the lattice. Obviously, it is always possible to extend this definition to $\mathscr{T}$.

We introduce a third dimension which represents a kind of gravitational potential, which shall be associated to each vertex v of the simplicial complex. We do this choosing a musical object $\mathscr{M}$ (a chord or a scale in this context) and computing the dissonance $d(\mathscr{M},v)$ of each note represented on $\mathscr{T}$ respect to this object. There exist several models to compute dissonance. Some of them has been tested. [See: Temperley, 1999, Plomp and Steeneken, 2005, Plomp and Levelt, 2005, Dillon, 2013].

Let $\mathscr{M} = \{p_1, \ldots, p_n\}$ be a set of pitches, and $S = \{v_1, \ldots, v_n\}$ the notes represented on $\mathscr{T}$. The height function $h : V \subset \mathbb{R}^2 \to \mathbb{R}^3$, maps the vertex $v = (x, y) \in \mathscr{T} \mapsto v^* = (x, y, d(\mathscr{M}, p))$.

If $\mathscr{M}$ is a C major triad and $\mathscr{T} = T(3,4,5)$ we obtain the complex depicted in figure 2.

Analysis

From a topological point of view the complexes we defined are always contractible. However their 0-skeleton can be thought as a point cloud in $\mathbb{R}^3$. A natural way of studying point clouds is persistent homology. [See: Verri *et al.*, 1993, Edelsbrunner *et al.*, 2002, Ghrist, 2008].

The components to investigate a point cloud via persistent homology are a filtration function, a good way to represent the life and death process of n-dimensional holes, and a distance to compare the results.

Different filtration functions have been used and information is represented both with barcodes and corner points diagram. [See: Carlsson and Zomorodian, 2009].

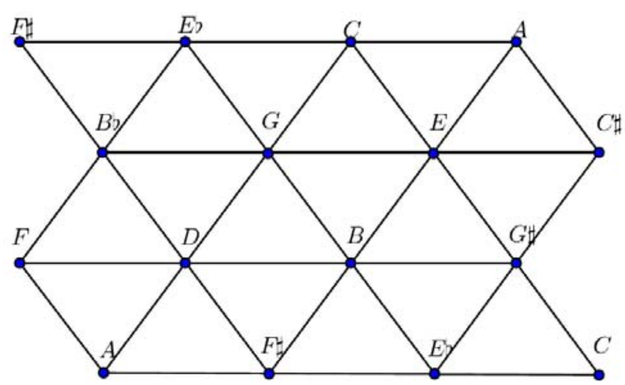

Fig. 1 The representation of a portion of $T(3,4,5)$ as a 1-skeleton of a simplicial complex.

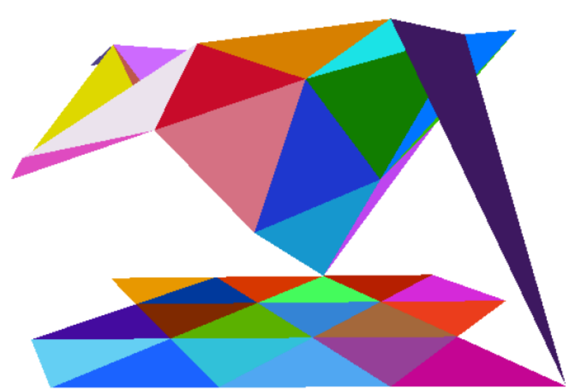

Fig. 2 The major triads on the Oettingen Riemann Tonnetz. In this picture triangles (2-faces) have been added to represents 3-notes chords.

results about the space of chords (see [Tymoczko, 2011]). Chords' space are deeply linked to configuration spaces, see figure 3 and 4. Thus it is natural to try to give an interpretation of voice leading in terms of braids. Since the fundamental group of $\mathbb{T}^n/\mathscr{S}_n$ is always equal to $\mathbb{Z}$, we made some investigation among different manifolds and quotient spaces to find the most suitable environment to represent voice leading through braids. In addition we give a representation of voice crossing, analyzing its importance in orchestration.

We also discuss some interesting geometrical remarks on Estrada's combinohedron. Partitions of 12 find an interesting representation in terms of simplices. For instance the orbits of the action of the symmetric group of order n on the n components partition of 12 are surprisingly relevant in terms of standard harmony: the augmented triad which is fixed by the action of $\mathscr{S}_3$ is the barycenter of a 2-simplex, it is surrounded by the hexagon of major and minor triads. This, in turn, is surrounded by a triangle of diminished triads. [See: Ramırez Alfonsin & Romero, 2002, Estrada, 1994].

Acknowledgement

These are the results of joint researches with Stefano Baldan, Moreno Andreatta (Tonnetz and Dissonance), Riccardo Jadanza, Alessandro Portaluri and Alexandre Popoff (braids approach to the space of chords) and Giulio Masetti (combinohedron's geometry).

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Special panel "Mathematics and Aesthetics in Julian Carrillo's (1875-1965) work":

"An Analytical-Comparative Approach to Carrillo's Metamorphosis and Wyschnegradsky's Non-Octaviant Spaces and their Reverberations"

Keywords: Julián Carrillo, Ivan Wyschnegradsky, Jean-Etienne Marie, microtonal pianos, musical metamorphosis, non-Euclidian geometry, non-octaviant spaces, third tone, sixth tone, twelfth tone

Abstract: Carrillo was a rather narcissistic person. He considered himself to be the greatest revolutionist of music history, as the first and only inventor of microtones and as the saver and multiplicator of European music. As with almost all narcissistic composers, Carrillo had no real followers. His theories did not make a great impact and his instruments, above all his 15 special pianos, have remained mute in Mexico for almost fifty years. By contrast, more than 80 pieces have been composed for the two instruments that Carrillo bequeathed to his French friend Jean-Etienne Marie. Especially the 16th-tone piano, with its possibility of realizing a glissando on the piano, has a particular fascination for European composers.

During Carrillo's lifetime only two musicians, far away from Mexico, were interested in his microtonal pianos: Jean-Etienne Marie and Ivan Wyschnegradsky. For the Russian born Wyschnegradsky living in Paris, the encounter with Carrillo's pianos at the World exposition 1958 in Brussels represented a creative choc. He had just developed his idea of non-octaviant spaces, a completely new concept in harmonic thinking implying revolutionary considerations of the musical space. The low volume of Carrillo's pianos in combination with their strong differentiation of intervals inspired Wyschnegradsy to enlarge his musical systems. He began to compose three pieces for the Carrillo pianos in third tone, sixth tone and twelfth tone in which he explored new musical realms.

In this lecture I will analyze how Wyschnegradsky improved his own compositional system, which was largely influenced by Non-Euclidian Geometry, in confrontation with Carrillo's concepts of musical *metamorphosis*. As friend of both Carrillo and Wyschnegradsky, Jean-Etienne Marie conciliates the two systems of metamorphosis and non-octaviant spaces thus sparking many of the reverberations that have influenced European music since Carrillos death in 1965.

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"Dynamical Virtual Sounding Networks: An Algorithmic Compositional Structure Based on Graph Theory and Cellular Automata"

Keywords: algorithmic composition, electroacoustic music, graph tehory, cellular automata, complex networks.

Abstract: In this work we will present a method for algorithmic composition based on a different approach than the traditional mapping found in most of actual artworks based on algorithmic processes, proposing a new perspective that includes a theoretical math background and a more complex mapping process. A Rhythmic Space R_S is defined as the set of rhythmic musical structures provided with an specific sum and multiplication operations. Elements from that space are then operated as real numbers with the rhythmic element quarter defined to be as the neutral multiplicative. This set shows no commutativity, neither distributive or associative laws under the operations previously defined due to the very inner musical properties of the rhythmic elements. In this way, rhythmic functions can be defined according to the set of operations and so, basic rhythmic structures can be transformed into a more complex process as a consequence of these functions.

As a practical application for composition, this paper shows a complete algorithmic process based on graph theory and cellular automata topics. An initial directed graph *C* of size n along with its adjacency matrix is defined, where each vertex is mapped to a previously defined rhythmic function so the graph turns into a dynamic complex rhythmic network. The adjacency matrix is then reinterpreted as a 1D or 2D cellular automata state, so, further transformations of this matrix can be performed using any rule at any time. A reconfiguration of the graph takes place each time and so multiples and different rhythmic structures are generated each time step.

As will be shown in the piece *Natividad* generated entirely by this model. This application is not restricted to rhythmic aims and once the general structure is defined, any sound transformation can be executed including pitch, timbrical properties, dynamic loudness, etc.

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"A Proposal for a Music Writing for the Visually Impaired"

Keywords: Braille music, transcription, music literacy.

Abstract: Braille Musicography is the most used system by blind people for reading and writing music in the world. It is a transcription from the conventional music notation to a system with symbols generated by a matrix of raised dots of two columns and three rows. It shows two difficulties which make it a hard tool for the blind musicians: 1) the number of music symbols largely exceeds the number of possible Braille combinations; and 2) it is a linear system representing a bidimensional system.

The main purpose of this work is to propose a set of symbols that will be the base of a system for reading and writing music for the blind people, as an alternative to Braille Musicography. Different types of symbols will be tested, among them symbols generated by a matrix different from the one with 3 lines and 2 rows which is generally used. It is important to notice that our fingertips have a delimited zone in which the density of receptors is high and allows a clear reading of a symbol. Outside this zone, the produced mental image is not clear and makes the reading tiresome and difficult. The symbols of the system proposed in a later stage of this work will have to respect these dimensions. Some other physiological and cognitive considerations have to be taken into account, in order to obtain a useful system. If music notation is conceived as a regular language, then it is possible to establish a relationship between conventional notation and Braille Musicography, being able to solve some of the difficulties of the last and making transcription more efficient and maybe totally automatic. Based in all these results, and considering the experience and ideas of the blind people who use music notation in Braille, a new system will be proposed, looking for a more efficient system than the actual Braille Musicography.

The main purpose of this work is to propose a set of symbols that will be the base of a system for reading and writing music for the blind, as an alternative to Braille Musicography. Different types of symbols will be tested, among them symbols generated by a matrix different from the one with 3 lines and 2 rows which is generally used.

It is important to notice that the most sensitive zone of our fingers is the fingertip. Two kinds of receptors, Meissner's corpuscles and Merkel's disk, are in charge of tactile acuity given their characteristics (Gardner *et al.*, 2000 : 431–437). There are more of these receptors in the fingertip than in the rest of the hand (*ibid.* : 437). The area where this happens is approximately 25 mm² (Detorakis, 2014 : 8). The density of these receptors increases drastically from the palm to the finger tips with two abrupt increases (Johansson, 1977 : 284), delimiting a very sensitive zone as shown in Figure 1. This allows a clear reading of a Braille symbol (Gardner *et al.*, 2000 : 435). Outside this zone, the fast reading of the symbol produces a blurred image, making the reading tiresome and difficult.

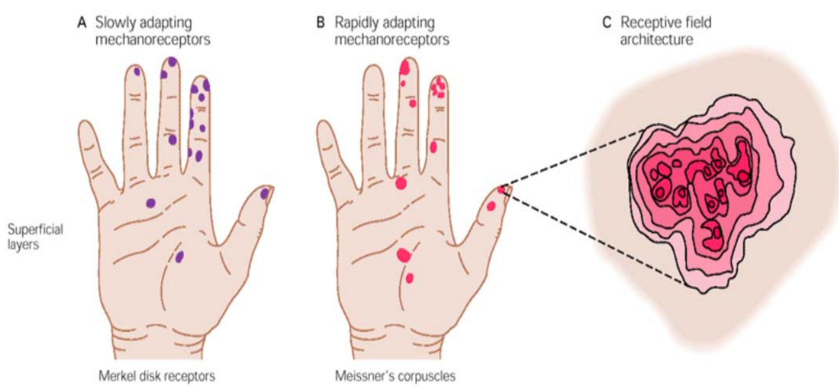

Figure 1. Taken from Gardner *et al.*, 2000 : 434.

The symbols of the system that will be proposed in future stages of this work will have to respect these dimensions in order to provide a useful system. The dimensions are respected by a standard Braille box as described in the document by BANA "Size and Spacing of Braille Characters".

If music notation is considered as a regular language, then it is possible to establish a relation between conventional notation and Braille Musicography, being able to solve some of the difficulties of the last and making transcription more efficient and maybe totally automatic.

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"Mazzola's Escher Theorem"

Keywords: Escher Theorem, topological categories, categorical gesture, digraphs, limit, colimit.

Abstract: Along the history of performance research there are two different unrelated approaches: the empirical and the philosophical one. From the empirical point of view, the quantitative aspects of performance are treated, and parameters need to describe performance as they are specied in a numerical way. Instead, from the philosophical standpoint, qualitative aspects and thoughts expressed through performance are treated. This two threads of performance research were disjoint during a lot of time up to 1992, when the research group of Guerino Mazzola started investigations about a general theory of performance.

The goal of this talk is to overview Mazzola's Escher Theorem for topological categories of hypergestures. This result states the existence of a canonical isomorphism

$$\Gamma\vec{@}\Delta\vec{@}K \cong \Delta\vec{@}\Gamma\vec{@}K,$$

where Γ, Δ are digraphs and K is is a topological category. During the talk, the concepts needed to prove the Escher Theorem will be detailed and reviewed, all of which are used in several research papers by Professor Guerino Mazzola. The theorem at hand is needed for the development of the theoretical apparatus that Mazzola seeks to realize, and it leads us to a better understanding of the concept of *categorical gesture*, as used in his work about a general (mathematical) Theory of Performance. The talk will be complemented with further comments on the Theory of Performance, from interviews to conductor Wilhelm Furtwangler, and conductor and composer Pierre Boulez.

Acknowledgement

This talk is a report of my undergraduate thesis under Lluis-Puebla's direction, which is based on two research papers of Guerino Mazzola.

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"The Mechanics of Tipping Points: A Case of Extreme Elasticity in Expressive Timing"

Keywords: expressive performance, timing and tempo, extreme elasticity, mathematical models.

Abstract: Musical timing forms the essence of expressive performance. Expressive timing serves to delineate structures and draw attention to musical features [5]. As in the case of stand-up comedy, the right timing can make the difference between a riveting performance and a lackluster one. As illustration of the importance of musical timing, a simple exercise can show that playing a piece with appropriately shaped timing, albeit with many wrong notes, is preferable to playing all the right notes with broken timing.

Research on expressive timing has centered on aspects of phrasing, which are primarily defined by a rise and fall in local tempo or dynamics. Repp [7] showed that these tempo phrase arcs can be described by quadratic functions; Repp [8] further demonstrated that transitions from one tempo to the next can be modeled by cubic functions. Kinematic approaches to

modeling tempo showed that a physical body coming to a stop better approximated ritardandi [3]. Taking the locomotive analogy a step further, Chew et al. [2] created a driving interface for the shaping of tempo trajectories.

While much work has focused on the ebb and flow of tempo that mark phrasing, little work addresses gestural forms of timing deviations, which can exhibit far more extreme degrees of elasticity. In 2010, Rajagopal observed that local tempo variations at the start of Gould's 1977 and Pogorelich's 1986 performances of Bach's *Saraband* (BWV 807) resembled a damped harmonic oscillator, thus suggesting that, beyond modeling beats and meter [4,6], oscillators can also be used to describe tempo fluctuations.

In [1], borrowing from physics, I introduced the tipping point analogy for musical timing. A musical tipping point is a massive distortion of the tempo, a musical hyperbole, which extends well beyond the normal pulse and meter. It can be defined as a timeless moment of suspended motion, beyond which a small perturbation will tip the balance and set in motion the return of the pulse. Relatively rare over the course of a piece, tipping points form the defining moments of a performance.

I shall describe the mechanics of these tipping points: how timing at tipping points can be executed and modeled mathematically, taking into account parameters of pitch and loudness; and, when tipping points can be employed—how they play on expectations both schematic (such as familiarity with tonal conventions) and veridical (such as knowledge of a well-known theme).

This work deepens existing, and makes concrete new, connections between music and motion. It raises the question of whether the music-motion link results from the use of movement metaphors to shape performance, or from tempo variations possessing the characteristics of low order differential equations.

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"Lexicographic Orderings of Modes and Morphisms"

Keywords: Christoffel words, combinatorics on words, lexicographic order, modes, Sturmian morphisms, well-formed scales.

Abstract: The first part of this paper will be a brief and, one hopes, gentle introduction to the theory of the modes of well-formed scales, within the framework of algebraic combinatorics on words, specifically musical modes encoded as members of the monoid of words A^* over a two-letter alphabet A, and the monoid of special Sturmian morphisms that act on A^*. This material was exposed for a music theory audience in Clampitt and Noll 2011 (*Music Theory Online* vol. 17.1).

The second part of the paper presents new work, relating lexicographic orderings of words encoding the modes of (non-degenerate) well-formed scales (especially the canonical examples, the diatonic modes) and lexicographic orderings of the special Sturmian morphisms associated with the modes, to the musical scale and circle-of-fifths orderings. In particular, departing from the 2013 Montréal SMCM paper by Noll and Montiel (in Springer LNCS vol. 7937), the lexicographic orderings are related to Zarlino's (1571) re-ordering of the six authentic Glarean (1547) diatonic modes and the circle-of-fifths folding words related to the authentic modes via the duality referred to in Clampitt and Noll 2011 as the twisted adjoint.

Scale Theory Concepts and Notations

Scale theory derives its mathematical character from the fact that musical scales are generally periodic phenomena. Most often the period is the musical octave (associated with the frequency ratio 2:1). One may then identify a given scale with a set of fundamental frequencies f_k, $1 = f_0 < f_1 < \ldots < f_{N-1} < 2$, N a positive integer, and, taking base-2 logarithms, with elements $0 = s_0 < s_1 < \ldots < s_{N-1} < 1$, where $s_k = log_2(f_k)$. We call the elements s_k the *scale steps*, and the differences $(s_j - s_i)$ mod 1 the *specific intervals*, or the *specifc interval sizes*. The specifc intervals $(s_{i+1} - s_i)$ mod 1 for $0 \leq i < N$, $i + 1$ reduced modulo N in the case $i = N - 1$, are defined to be the (*specific*) *step intervals*.

Already the existence of periodicity and other possible symmetries suggest the application of mathematics: the Discrete Fourier Transform, for example, or group theory models. Both approaches have been fruitful; here we begin with very constrained groups (the building blocks of abelian groups), moving to very unconstrained non-abelian groups (free groups with two generators), and to free monoids and their morphisms. The first candidate is the cyclic group modulo N.

The first conceptual step in scale theory is to identify the scale steps with their index numbers, i.e., with the mod N residues $\mathbb{Z}_N$, and define the *generic intervals* by differences between index numbers, again the set $\mathbb{Z}_N$, which may be understood to be an additive group with addition modulo N, $\mathbb{Z} = N\mathbb{Z}$. We also call these numbers the *lengths* of generic intervals. The generic intervals associated with length 1 are the generic step intervals, and the generic interval lengths capture the notion of scale-step measure, as in the musical nomenclature "2nds, 3rds," etc. A double description of musical intervals via generic and specific intervals is the approach to scale theory taken by a number of theorists, initiated formally by Clough in [1] and continued by Clough and Myerson in [2]. Scales in which each non-zero (mod N) generic interval corresponds to a fixed number of specific interval sizes are of particular musical interest. The cases where each generic interval corresponds to one specific interval, i.e., where the generic/specific distinction is collapsed, are equal divisions of the octave. The cases where each non-zero generic interval corresponds to exactly two specific interval sizes are a privileged class which are shown in [2] to be equivalent to scales having the Cardinality Equals Variety property, and are shown in [3] to be equivalent to non-degenerate well-formed scales (to be defined below). The class of scales for which each non-zero generic interval corresponds to three specific intervals is a much larger one, but it is shown in [4] that all pairwise well-formed scales belong to this class. We will be concerned here with the second type, the non-degenerate well-formed scales.

Let $S = \{s_k | 0 = s_0 < s_1 < \ldots < s_{N-1} < 1\}$ as above. If there exists a real number $0 < \theta < 1$ such that for each k from 0 to $N - 1$ we have $s_k = (n\theta)$ mod 1 for some $0 \leq n \leq N - 1$, we say that S is generated, and θ is a *generator* for S.

By the pigeonhole principle, we have one of the elements $s_j = 1 \times \theta$, so one of the specific interval sizes in the scale S is the number that generates S; we will refer to it as the *generating interval*. It follows that $1 - \theta$ is also a generator for S whenever θ is. Let S be generated with generator θ. We may then write $S = \{0, (n_1\,\theta) \text{ mod} 1, \ldots, (n_{N-1}\,\theta) \text{ mod } 1\}$. Then S is defined to be a well-formed scale if the mapping of index numbers of elements of S onto the elements n_i is an automorphism of $\mathbb{Z} = N\mathbb{Z}$.

That is, S is well-formed if there exists a fixed integer m, $0 < m \leq N - 1$, such that $\Omega : \mathbb{Z}\ /\ N\mathbb{Z} \mapsto \mathbb{Z}\ /\ N\mathbb{Z}$: $z \mapsto n_z \equiv mz$ (mod N). We say that S is *degenerate well-formed* if all of the step intervals are of the same size; *non-degenerate well-formed* otherwise. But here we will take well-formed to mean non-degenerate well-formed.

From this definition flow a series of equivalences and entailments, relating the cardinality of the scale (N), the multiplicities of the step intervals (p and $q = N - p$), and the automorphism Ω. Carey and Clampitt showed in [5],[6] that the cardinalities of well-formed scales are denominators of (semi-)convergents in the continued fraction representation of the generator θ. (NB: Semi-convergents are also called *intermediate convergents*; best approximations from one side.) There is thus a hierarchy of well-formed scales of increasing cardinalities, finite if θ is rational, infinite if θ is irrational [5]. As noted above, S is non-degenerate well-formed if and only if all of its non-zero generic intervals are associated with two specific interval sizes. In particular, the step intervals come in two specific sizes, a and b, with multiplicities q and p, respectively, which are coprime with N [3]. It is demonstrated in [3] that the generic length of a generating interval θ (or $1 - \theta$) is the multiplicative inverse of one of the step multiplicities. That is, if $\theta = s_j$, then as a generic interval it has length j, and we have $jp \equiv 1$ (mod N). Which of the multiplicities p or q is the multiplicative inverse and which the negative of the multiplicative inverse depends on whether N is the denominator of an even or odd (semi-)convergent to θ in its continued fraction representation. It follows that the value m that defines the characterizing automorphism Ω is p or its negative, depending again on whether N is the denominator of an even or odd (semi-)convergent.

Since all scale structure is determined by the integer values N and p, the next step in the theory is to consider equivalence classes of well-formed scales. Let WF(N, p) be the class of all well-formed scales with cardinality N and multiplicity p of specific step interval b. It is evident that these classes partition the universe of well-formed scales and are thus equivalence classes. A representative of WF(N, p) may be taken to be a string of a's and b's representing the step intervals in S, that we call a *step-interval pattern*. For example, the usual diatonic scale, belonging to the class WF(7,2), represents the class with the step-interval *pattern string*, or "word", *aaabaab*.

As a mode of the usual diatonic scale, this corresponds to the Lydian mode, where a represents a whole step and b a half step. As a white-note scale, it would be the mode on F: F G A B C D E (F'). This exemplifies the *canonical* step-interval pattern, when S is in the form of a generated set. This is the form the usual diatonic would take if it were presented as the generated set S with generator "perfect fifth."

We might take the interval size of the perfect fifth to be $log_2\frac{3}{2}$, corresponding to the frequency ratio 3/2, but for ease of computation, let us take the equal-tempered perfect fifth, $\frac{7}{12}$. Then S has the form S_{ETdia} = {0, 1/6, 1/3, 1/2, 7/12, 3/4, 11/12}, where the elements are sk = $((2k) \bmod 7)(\frac{7}{12} \bmod 1)$, for $k = 0, 1,\ldots, 6$.

We now turn our attention from well-formed scales to their modes, that is, cyclic permutations of step-interval patterns. These may be understood to be derived by shifting the initial point s_0 of S to another element of S. For example, if in the usual diatonic in its canonical Lydian mode form, as above, we subtract the value of the generator $\frac{7}{12}$ from each value of S_{ETdia}, we have the familiar major scale form, {0, 1/6, 1/3, 5/12, 7/12, 3/4, 11/12}, that gives rise to the new step-interval pattern *aabaaab*. Now we consider each class WF(N, p) to include and have as representatives the cyclic permutations (rotations, conjugates) of the canonical step-interval pattern. It is at this point that algebraic combinatorics on *words* (word theory) becomes a useful perspective.

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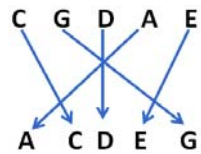
"Set Theory and its use for Logical Construction of Musical Scales"

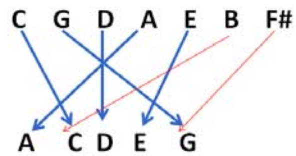
Abstract: A lot is said about the relationships between music and mathematics. However very few times those relationships are given in clear and applied cases. At the Luthiers School (Escuela de Laudería – INBA, Querétaro, Qro.) these connections between science and art have been useful. Understanding mathematics and music theory as a building theory that shows links and connections with art and science implies understanding lutherie as an applied art, related to science and history. At the beginning of the career, the relationships between science and music stands out as part of instrument design, involving therefore theories of physical shapes and mathematical resources. In this field I identified the easiest way for students to understand set theory and mathematical functions related with music theory. Another case is by building modal scales and their relatives by the function previously given; fine connections can be made by associating major scales with their minor relatives.

The aim of this proposal is to point out essential matters in which set theory and mathematical functions match with music theory as a proposal to grant holistic knowledge of music and mathematics. In set theory "a function or map from X to Y is an association between the members of the sets. More precisely, for every element of X there is a unique element of Y" (Houston, 2009). By simple analogy we can explain relative scales in set theory, as we define any musical scale as a given set. In traditional Western tonal music, one set may represent major scales and another set, minor scales. The way or *map* to relate a major scale to a minor scale, is our function. This map is needed to go from the minor scale to the major one, and for that we have to take the third sound of our minor scale (*third grade*) and use it as the first grade of the major scale. According to tonal tradition, what makes a major scale is the distance between two tones from the first grade to the third. In the minor scale the distance is of one and a half tone. But if we think on the relative scales as subsets, then we have a different kind of relationship and a different map to relate them.

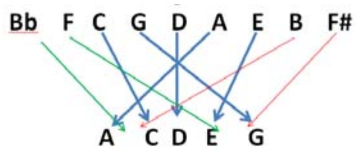
Using the easiest way to explain the diatonic scale without using too much musical theory, we will start in C to build, by fifths a five sound scale, the so called pentatonic scale: C G D A E. These letters (*pitches*) are ordered without considering if we passed an octave, so the set is of the seven pitches scales built by perfect fifths. Let us find out how we can build the modal scales by perfect fifths. Transposing a few sounds an octave to order them in a simple octave, we have:



We can go further adding two more perfect fifths in three different ways to the original five letters we use to get our pentatonic scale, by (1) adding the two fifths on the right (higher); (2) adding the two fifths on the left (lower); (3) adding one fifth to the right and one to the left. Following this method, we build the six modal scales which will relate by a specific function. For the first case, which is to add two perfect fifths to the original five pitch scale used to build the pentatonic scale, higher to the five tones, we have—ordering them and adding the octave (A´)—the Dorian mode A BC D E F#G A’ by the function



For the second case, adding the two perfect fifths on the left, in descendent way from the lowest pitch and ordering them in the same octave, Ab*b* C D EF G A’ by this function resulting in the Phrygian mode:



A third mode is built by adding a perfect fifth higher and a perfect fifth lower to our original 5 pitches from where we got the pentatonic scale. The result is the Aeolian mode A BC D EF G A’. This is the set of the scales made of perfect fifths.

Can we either think of these scales as two different sets (the major scales set, and the minor scales set), or should we consider them as a subset one from each other? In music theory this point of view would provide an advance for understanding tonal music as a function of shifts from *major* to *minor* (or vice-versa), or as a whole scale function with different grammars for its grades and tonal relationships. Thus let’s try to think of set theory as a way we will take for understanding music theory.

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“Music of Quantum Circles”

Keywords: circle, symmetry, quantum geometry, diagrammatic category.

Abstract: We illustrate the basic ideas and principles of quantum geometry by considering mutually complementary quantum realizations of circles. It is quite amazing that such a simple geometrical object as the circle, provides a rich illustrative playground for an entire array of purely quantum phenomena. On the other hand, the ancient Pythagorean musical scales naturally lead to a simple quantum circle. In this lecture we explore different musical scales, their mathematical generalization and formalization, and their possible quantum-geometric foundations. In this conceptual framework, we outline a diagramatical-categorical formulation for a quantum theory of symmetry, and further explore interesting musical connections and interpretations.

1. *Introduction*

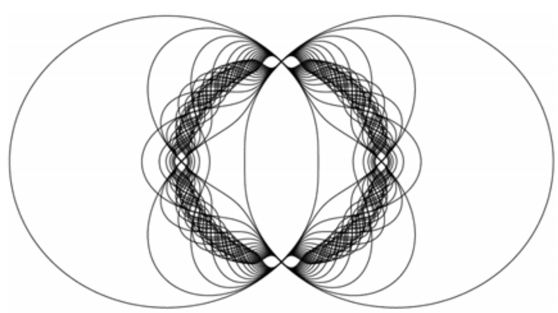
Quantum geometry mirrors the ideas of quantum physics, into the realm of geometrical spaces and their transformations. But quantum spaces, the analogies of atoms, mollecules and quantum systems of physics in general, exhibit a nature essentially different from their classical counterparts. They are not understandable in terms of points, parts, or local neighbourhoods. In general, these concepts do not apply at all to quantum spaces. However the entire fabric of space is conceived as the one indivisible whole.

There is something profoundly quantum in all music. A discrete space–the skeleton hosting any musical score, morphs into a true musical form, only after being symbiotically enveloped by a *geometry of sound*. And this geometry is inherently quantum, as it connects the points of the discrete underlying structure, invalidating the difference between *now*, *then*, *here* and *there*; thus creating an irreducible continuum for a piece of music: continuous discreteness and discrete continuity.

All this inherently promotes *simplicity* in thinking, as we are forced to look for some deeper structure, going far beyond the *parts*, *points*, *local neighbourhoods*, and fragmented classical geometrical views. One such a way of thinking, transcending the nature of mathematical realms, is *harmony*: to look at symmetries—the transformational modes of things—and understanding the mathematical creatures in terms of them. Conceptual roots of this thinking are found in the *Erlangen Program* by Felix Klein.

Circles are children of simplicity. A principal geometrical realization of an infinite symmetry group. The idea of circle is observed in *repetitions*. Any continual change, movement, transformation, in which there is something invariant before and after, naturally leads to the idea of circle. In music such is the concept of *octave*. It leads to a circle representing the geometrical space of abstract tonalities. A more detailed geometrical structure is given by a *musical scale*, interpretable as further 'musical' symmetries of the circle.

The aim of this lecture is to illustrate how these symmetries of the circle lead to its own projected quantum realizations, and the complementary view of extending the circle into a quantum counterpart. These examples are actually extremely rich in their internal structure. They reflect the spectrum of all principal new phenomena of quantum geometry. In particular, the quantum circles are *quantum groups* in a proper sense. We shall briefly talk about a general diagrammatical and categorical formulation of symmetry, which naturally includes our quantum circles and their quantum siblings, as well as the variety of all classical structures.



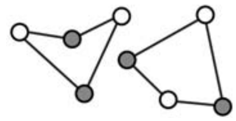
2. *Quantum Circles*

The Pythagorean musical scale invites us to consider the quotients of the classical circle $\bigcirc$ over a free action of the infinite cyclic group of integers $\mathbb{Z}$, generated by a single irrational rotation. The space of equivalence classes has a direct musical interpretation, as the space of abstract tonality classes within a single octave. In the case of Pythagoreans, we have two principal frequency transformations: the *octave* itself, given by doubling the frequency $\omega \mapsto 2\omega$; and the *perfect fifth*, given by the shift $\omega \mapsto 3\omega$. If we consider the frequency range as covering all positive real numbers $\mathbb{R}^+$, and pass to natural logarithms, then the multiplication becomes addition and frequency range is the whole $\mathbb{R}$. The octave space is given by $\mathbb{R}/\mathbb{Z}\ \ln(2)$. Within this space, the addition of $\ln(3)$ acts as a symmetry. By transforming $[r] \mapsto \exp(2ir\pi/\ln(2))$ we can identify the octave space with the circle $\bigcirc$ of the unitary complex numbers. In terms of this identification, the Pythagorean perfect fifth becomes a multiplication by $\exp(2i\pi\ \ln(3)/\ln(2))$ which represents an irrational rotation, by the angle $\varphi + 2\pi = 2\pi\ \ln(3)/\ln(2)$.

Another possibility is to consider rational rotations. In terms of complex numbers, it corresponds to roots of unity, say primitive solutions of the equation $z^n = 1$ for $n \geq 2$. In this case the action of $\mathbb{Z}$ factorizes to the action of the cyclic group of order n on the circle. And the resulting factor space is again a classical circle. So our tonality space is given by an n-fold covering of $\bigcirc$ by $\bigcirc$. Musical scales based on equal temperament provide a realization of such a rational structure, and n is the number of semitones. In terms of the original frequencies, the simplest movement is given by $\omega \mapsto 2^{1/n}\omega$

However, in the irrational case, there exist infinitely many connectable pitch values, dense in the octave space. In other words, every orbit of the action is dense in the circle $\bigcirc$. The resulting orbit decomposition is ergodic in the sense that there exist no no-trivial decomposition of the circle, into two disjoint measurable sets consisting of whole orbits each. One of them always has measure 0 and hence another is of the normalized measure one. To put it differently, there exist no no-trivial measure theory on the orbit space Q. It exhibits a kind of intrinsic *wholeness*. And if there is no measuring in Q, then there is simply no hope to build, in the spirit of classical geometry, any meaningful higher-level theory.

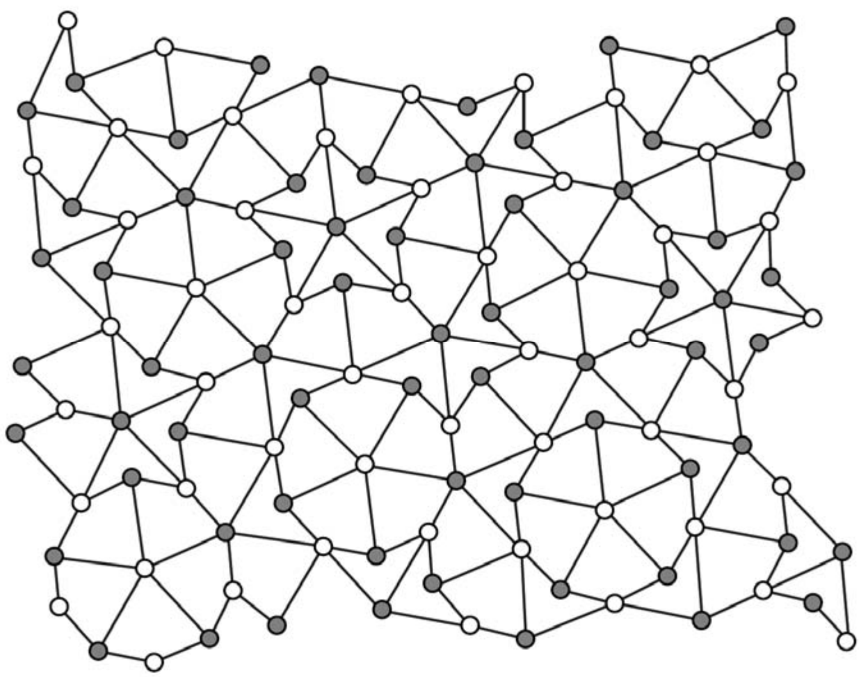
So Q is consisting of points, however the points are behaving quite wildly, and there is no any effective and operational separability between them. Exactly the same kind of phenomenon we encounter in studying certain *aperiodic tilings* of the Euclidian plane. A paradigmatic example is given by the space of isomorphism classes of Penrose tilings. There exist (uncountably) infinitely many classes, however every two tilings are indistinguishable by comparing their finite regions. Every finite region of one tiling is faithfully echoed, and infinitely many times in any other tiling.



This space can also be described as the quotient space of the full binary sequences space $\{-, +\}^{\mathbb{N}}$ which is the same as the Cantor triadic set, by a relation of equivalence identifying sequences which coincide on a complement of a finite subset of $\mathbb{N}$.

One possibility to deal with such quantum points, is to construct a noncommutative C*-algebra A, which captures the space Q in terms of equivalence classes of its irreducible representations. Such an approach is presented in detail in [1]. Another and inequivalent approach, is to apply the theory of quantum principal bundles developed in [2], and consider non-trivial differential (necessarily quantum, as in music) structures on discrete and extremely disconnected spaces and groups. We believe this is more in the spirit of the original Erlangen Program.

So the rational rotations give us classical circle as the tonality classes space. And irrational rotations produce quantum objects. It is interesting to observe that from a purely geometrical perspective, the quantum behaviour is *the generic one*. Indeed, although the rational and irrational unitary complex numbers are intertwined, both being everywhere dense in the circle, the roots of unity form a countable and therefore negligible, subset. With probability one, $\bigcirc$ will choose an infinite covering mode, and cast a quantum shadow.



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"A Group for Pitch Sequences Representation with Emphasis in Debussy's Music"

Keywords: Group Theory, intervals, concatenation, Debussy, pitch sequences.

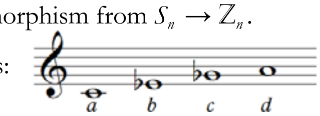
Abstract: Through the use of archaic modes and whole-tone or pentatonic scales Debussy cultivated a musical style known as *impressionism*. Here we offer an intuitive way to represent whole-tone musical phrases as elements in impressionist music in mathematical terms, as well as other musical phrases with regular intervals elements using a set we named $L(S_n)$.

Fix $n \in \mathbb{N}$. Let S_n be the set whose elements s_j, $j \in \{0, \dots, n-1\}$, are sets of intervals of $\frac{12}{n} j$ semitones, including its octaves; in other words, $s_j = \{\dots, -12 + \frac{12}{n} j$ semitones, $\frac{12}{n} j$ semitones, $\frac{12}{n} j$ semitones, $\dots\}$. Then we have the sets S_1, S_2, S_3, S_4, S_6 and S_{12} whose elements are equivalence classes. We shall name elements in S_n using letters in ascending order starting from the letter a, $S_1 = \{a = [0]\}$, $S_2 = \{a = [0], b = \left[\frac{12}{2}\right]\}$, …, $S_6 = \{a = [0], b = \left[\frac{12}{6}\right], c = \left[\frac{12}{6}2\right], d = \left[\frac{12}{6}3\right], e = \left[\frac{12}{6}4\right], f = \left[\frac{12}{6}5\right]\}$.

Now we define the operation + as the usual modular arithmetic, that is $[x] + [y] = [x + y]$, e.g. $a, b, f \in S_6$, $b + f =$ [2 semitones + 10 semitones] = [12 semitones] = a . We see that $(S_n, +)$ is a group with a being the identity element.

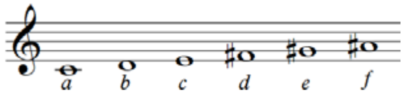
Now lets define $g : S_n \to \mathbb{Z}_n, g(s_j) = [j]$, $j \in \{0, \dots, n-1\}$; it is clear that g is an isomorphism from $S_n \to \mathbb{Z}_n$.

As an example we show S_4 using middle C (i.e. C_4) as reference for counting intervals:



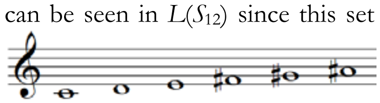
where each bar corresponds to each element in S_4.

Let $L(S_n)$ be an infinite set of infinite strings with elements in S_n concatenated in every possible order, also, each string has an infinite string of only a to the right; that is, for S_2, $a\bar{a}$, $b\bar{a}$, $ab\bar{a}$, $\in L(S_2)$ but also $babbabab\bar{a} \in L(S_2)$. We use over line notation to indicate repeating and never ending a. For convenience we will not write the infinite a string that goes with every element in $L(S_n)$, this way $babbabab\bar{a}$ will be just $babbabab$, also $a\bar{a}$ will be just a. This way we can represent pitch sequences as elements of $L(S_n)$; that is representing the movement of the melody by sequences of musical intervals. For instance, the phrase



is seen as the element $abab$ in $L(S_2)$.

We note that having $\bar{a}$ to the right does mean nothing to music since it is the identity element in S_n concatenated infinitely times. It does not add intervals. It is trivial to note that every sequence of sounds, as long as it uses some or all of the 12 pitches (disregarding enharmonics) in Western music, can be seen in $L(S_{12})$ since this set includes all possible sequence of intervals. This is how $abbbbb \in L(S_6)$ is seen in a staff:



This is the whole-tone scale starting at C_4, the other whole-tone scale can be generated in reference to $C\sharp_4$, every possible sequence of sounds using this scales can be seen as an element of $L(S_6)$.

As an example, we take a look at the first two bars of *Prelude No. 2*, *Voiles*, from first book of preludes by Claude Debussy:

* Ernest-Guter Heinemann. *Debussy Préludes, Premier Livre.* G. Henle Verlag, 1986.

We can represent the upper melody in reference to C_4 as $effffaf \in L(S_6)$ and the lower melody as $cffffff \in L(S_6)$. In this piano piece almost every phrase is an element of $L(S_6)$. Whole-tone elements are present in much of Debussy's repertoire. Just to mention few examples: everything from *Voiles* except 6 bars; the solo between english horn and cello at the end of the first movement in *La mer*; a number of passages in *Les images*, livre I, for piano solo.

Let s, $\grave{s} \in L(S_n)$, $s = [s_1][s_2] \ldots [s_n] \ldots$, $\grave{s} = [\grave{s}_1]\ [\grave{s}_2] \ldots [\grave{s}_n] \ldots$ Now we define the $\circ$ operation as a coordinate-wise addition in the sense of $s \circ \grave{s} = [s_1 + \grave{s}_1]\ [s_2 + \grave{s}_2] \ldots [s_n + \grave{s}_n] \ldots$ We note that the length of s and $\grave{s}$ does not matter since every element in $L(S_n)$ has $\bar{a}$ to the right, and this means there will always be an a to operate.

For the last example we look at the first beat of bar no. 31 in *Jeux d'eau* for solo piano from Maurice Ravel:

Using ($L(S_6)$, $\circ$) in reference to C_4, the the upper melody performed with the right hand, A♯ A♯ F♯ G can be represented as the *faeb* element in $L(S_6)$; now we arbitrarily select *face* and operate *faeb* $\circ$ *face*, and we obtain *eaaf* wich is the second beat in the same example:

Rafael Joseffy. *Jeux d'eau*. G. Schirmer, 1907.

Now we represent the upper melody in right hand from bars 31 and 32 (the given examples) of *Jeux d'eau* as follows: bar 31, beat 1: *faeb* in reference to C_4; bar 1, beat 2: *faeb* $\circ$ *face* = *eaaf*; bar 31, beat 3: *eaaf* $\circ$ *baec* = *faeb*; bar 31, beat 4: *faeb* $\circ$ *face* = *eaaf*; bar 32, beat 1: *eaaf* $\circ$ *baac* = *faef*; bar 32, beat 2: *faef* $\circ$ *faac* = *eaeb*; bar 32, beat 3: *eaeb* $\circ$ *faaa* = *daeb*; bar 32, beat 4: *caec* in reference to C$\sharp_4$.

Ravel, *Jeux d'eau*, bars 31 and 32.
Rafael Joseffy. *Jeux d'eau*. G. Schirmer, 1907.

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"Tone Rows and Tropes"

Keywords: Tone rows, trope, database, group actions, normal form, stabilizer type, transversal of non-equivalent objects, 12-tone series.

Abstract: Applying different methods based on group actions we provide a complete classification of tone rows in the twelve tone scale. The main objects of the present paper are the orbits of tone rows under the action of the direct product of two dihedral groups. This means that tone rows are equivalent if and only if they can be constructed by transposing, inversion, retrograde, and/or time shift from a single row. We determine the orbit, the normal form, the stabilizer class of a tone row, its trope structure, diameter distance, and chord diagram. The database contains complete information on all 836,017 pairwise non-equivalent tone rows. Bigger orbits of tone rows are studied when we allow further operations on tone rows as the five-step, the quart-circle or the exchange of parameters.

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Special panel "Mathematics and Aesthetics in Julian Carrillo's (1875-1965) work":

"Julián Carrillo's Numerical Notation in his Guitar Music: Challenges as an Interpreter and Performer"

Keywords: microtonality, Sonido 13, notation, performance, organology.

Abstract: Julián Carrillo's music, writings on music theory, compositions and organological innovations designed for diverse instruments have certainly been great contributions to the music of Mexico and the world. However, the study of his music and musical environment has been poorly approached so far, especially in the instrumental practice. The reformation proposed by Carrillo to simplify the writing system is based on eliminating the patterns, notes, accidents and keys of Western traditional music. Thus he proposes a horizontal line and two dashes, one above the line and another below, plus twelve numbers from zero to eleven to write music on the twelve-tone chromatic system. In the case of his microtonal music for guitar it must be said that, although the reading from topographical notations (such as tablatures of Renaissance and Baroque music) approaches the instrumentalist to Carillo's system, this presents specific challenges posed in many ways for those who want to play his repertoire.

The main goal of this proposal is to make a general review of the microtonal guitar repertoire written in the number system developed by Carrillo, as well as to present some solutions to technical and interpretive challenges of the Sonido 13 writing.

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"Partitiogram, *mnet*, *vnet* and *tnet* : Embedded Abstractions Inside Compositional Games"

Keywords: partitional analysis, Theory of integer partitions, Young's lattice, musical composition, musical analysis.

Abstract: This paper integrates a broad research about the pragmatic modelling of compositional process, and some mathematical abstractions that arises from the composer's decisions. *Partitional Analysis* (*PA* – Gentil-Nunes & Carvalho, 2003) is an original proposal of mediation between mathematical abstractions derived from the Theory of Integer Partitions (Euler, 1748; Andrews 1984; Andrews & Ericksson, 2004) and compositional theories and practices. Its main goal is the study of compositional games and has been used in the pedagogy of composition and in the creation of new pieces by professional composers as well as by students of composition. Some remarkable advances have been also achieved in musical analysis: the production of analysis of pieces from various historical periods, showing a significant convergence between the partitional structures and others analytical results (Gentil-Nunes, 2009, 2013; Santos, 2014; Monteiro, 2014). Analyses were also greatly facilitated by the implementation of the software *Parsemat®*, programmed by the present author, which streamlines the calculation plotting of graphics generated from MIDI files.

The point of departure of PA, inspired by the work of Wallace Berry (1976), is the consideration of binary relations between the agents of a musical plot. These agents can be any sound sources, qualities or elements used by the composer, for instance, voices, lines, timbres, positions, fingers, among others. The relations are categorized in collaboration and contraposition types, according to a given criterion (congruence between time points and duration, belonging to a line inside a melody, proximity of timbre or orchestral group, spatial location in the stage, and so on). This categorization in fact underlies the cognitive constitution of the partitions and, at the same time, leads to the establishment of the agglomeration (*a*) and dispersion (*d*) indices, each one corresponding respectively to the amount of collaboration and contraposition relations founded in a specific configuration. Plotting of one index against the other results in a phase space called *partitiogram*, where the musical progressions evaluated from the chosen parameter form a trajectory. The partitiogram constitutes an exhaustive taxonomy of all possibilities of action available for the composer inside that specific field. It shows also the kinship of its elements, evidenced by the metrified distances between the locations of partitions. According to the selected criterion, a distinctive application is established, leading to the constitution of a mapping of semantic kinds of actions or configurations. For example, the *rhythmic partitioning* is an application that observes the congruence between time points and durations of concurrent voices. The result is a mapping of all textural possibilities available to the composer, with increasingly massive textures distributed along the horizontal axis and increasingly polyphonic textures along the vertical axis. As the classical textures types, like monophony, heterophony, polyphony and homophony can be found scattered in the plane, many others are also represented, including some more radical examples founded in textural music from avant-garde period. On the other side, considering the relations between pitches and internal lines of a melody, for example, will lead to another application of

PA, the *linear partitioning*. In this case, the partitiogram reflects the general behavior of a melodic structure, including some known situations, like *arpeggiations*, lines and compound melodies.

The structure of partitiogram has an intimate affinity with Young's Lattice, a mathematical abstraction introduced in the early twentieth century by Alfred Young. Young was a British clergyman and mathematician who formulated the Young's Tableaux (representation of a partition, similar to Ferrers diagrams) and the Young's Lattice. The latter is a partially ordered set formed by all integer partitions, ordered by inclusion relationships, and ranked according to their sums. It can also be represented as a Hasse diagram. The internal structure of the partitiogram is in fact a metrified version of a Young's Lattice. Once the trajectories can give some information about the characteristic procedures of the composer, and once they share the same framework that underlies any possible compositional choice inside a specific field, it is also possible to create, as above mentioned, perfect and comprehensive homologies between musical elements that are not directly related, like texture, melody and timbre. Inside *PA*, the relations between adjacent partitions are qualified according to the nature of the specific progression. Three simple adjacency operators emerge from this procedure: *resizing* (*m*), *revariance* (*v*) and *transference* (*t*). *Resizing* involves unitary change in the size of one part (tapering or fattening). *Revariance* points to the addition or subtraction of a unitary part (changing in diversity of global content inside partition). *Transference* occurs when both operations (*m* and *v*) come into play, but with opposite signs, in a complementary way, thus preserving the total sum. Resizing and revariance are categorizations of inclusion relations, taking some features in account for musical purposes. Simple operators form networks of adjacency relationships, each one with specific profile and structure. Plotting the relations in computational applications leads to three functions – *mnet*, *vnet* and *tnet*, which basically analyze the relations between partitions for a given numbers of factors and draw the requested networks inside the partitiogram. *Mnet* has a fractal structure that exhibits curves and bifurcations in each of its iterations. *Vnet* has a more vertical and predictable distribution. *Tnet* is delineated by straight diagonals, each one linked to the number of elements involved.

The superposition of the three basic networks can represent the fundamental field of action of the composer and can also give rise to new applications, like typologies of compositional procedures, styles, and fingerprints. Using the partitiogram network as a pedagogic tool can lead to some kinds of creative games, including the application of canonic operations and transformations that can be the basis for compositional planning and can also be combined with other algorithmic techniques, like set theory, fractal development or progressive variations. The raised appliances can give to the student some conscience about their own work and its relation with all others possibilities, giving rise to new behaviors and consequently the expansion of its resources. Besides these basic functions, other operators, like *compound transfer*, *concurrence* and *regglomeration*, are being also investigated to become the basis for new analytical tools. All functions were developed inside Matlab environment and are integrated in the software *Partitions®* for Windows, programmed by the present author.

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Special panel "Mathematics and Aesthetics in Julian Carrillo's (1875-1965) work":

"Digital Technology in Julian Carrillo's Microtonal Music Research"

Keywords: digital technology, microtonal music, programming language.

Abstract: Many researchers that have tried to find the awakening of musical sounds agreed that Archytas solved the two mean geometrical problem or the musical interval three–parts division about 24,5 centuries ago [see: 1]; so he contributed to the Pythagorian notion of two–parts interval division [2]. However technology used to build musical instruments as well as music

forms used by composers in Western tradition, habituated ourselves to listen in the well known diatonic-chromatic system in which most of musical compositions and "master pieces" are composed and performed. This kind of music is based on the mathematical model $\sqrt[12]{2}$ or $2^{1/12}$ (Carrillo, 1956). Eventually, many musicians who lived in different times and geographies, tried to go on with Archytas developments; between them Ferrucio Busoni and Julian Carrillo. The former one obtained a musical interval division in fourth part, eighth part, sixteenth part, and so on, of tone when he did his practical experiment in 1895 [see: 11].

This lecture is based in the use of computational systems as a musical instrument that performs music compositons written in third parts, fourth parts, fifth parts of tone. To do this, a programming language allows the user to interact with the computer's loudspeakers, listening the so called *microintervals*. BASIC is used as programming language to write the software and to execute all melodic, armonic and rythmic transformations that are applicable to musical compositions (Carrillo, 1949). BASIC programming language has advantage over another ones because it is very economic in terms of hard disk space, use of mathematical language to write algorithms, and easiness to choose sound frequencies (for instance, 256 Hz or 261 Hz or any other value for central C). It also allows the use of the mathematical expressions $\sqrt[12]{2}$, $\sqrt[18]{2}$, $\sqrt[24]{2}$, $\sqrt[48]{2}$ [see: 9]. We show the program's involved instructions and execute several parts of Carrillo's compositions.

The final purpose of this contribution is focused on using digital technology as an option to listen microtonal music examples without disregarding some disadvantages of this techonology. We also discuss how string bows and attack in wind instruments are neither physically easy to be modelled nor they are tempered (contrasting Carrillo, 1930); as well as how the propositions commonly used to describe some musical phenomena, are not necessarily—or not always—based in terms of physics' *scientific language* (contrasting Carrillo, 1967). In spite of this, we stress the fact that our computer system and its programming language can be fruitfully used as a tool for research and education in microtonal music.

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"Representing Body Gestures Through Sound: Strategies for Mapping Body Gestures Focusing on the Transmission of Meaning Through Electroacoustic Music Sounds"

Keywords: body, gesture, kinetics, meaning, electroacoustic music.

Abstract: In this proposal I expose strategies that have been followed in order to represent the movement of the human body using vectorial representations of the position of its joints through time, yielding information with which I can measure their instant velocity and acceleration in real time.

Considering body from a self-referential perspective, we can represent gestures as the relationship between the trajectories drawn by the points that represent the joints through time. The focus of this work is set on mapping strategies that generate an environment that can represent body gestures as sounds, preserving the expressive properties of the first.

Introduction

The motivation behind the development of mapping strategies that can interpret a movement as a concurrent unfolding sound, lies on the creative restlessness of creating an environment that can promote both a synesthetic and an aesthetic experience. This particular focus on the objective of this work moves towards a wide understanding of music which can include the gestural experience of sound making as a pool of words that conform a vocabulary with which something can be transmitted. A code that is cyphered on gestures, a language common to all humans that needs no translation if it is to be considered universal, as the works of Ekman and Frisen suggest.

Considering gestural sound making has driven this research towards a physics representation of movement that takes advantage of the commercially available device Kinect to describe the parameters of movement at the actual moment in which a trajectory is being traced by the performer. These parameters are: position, instant velocity and instant acceleration.

All the parameters are given in 3 dimensions and there is a position for 14 different body parts to which I will refer to as joints: a head, 2 shoulders, 2 elbows, 2 hands, a torso, 2 hips, 2 knees, 2 feet. So, given a position $p[t] = (x, y, z)$ the velocity $\boldsymbol{v}$ of a joint can be calculates as

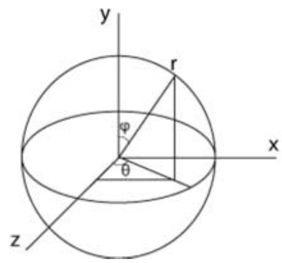
$$v[t] = p[t] - p[t-1]. \quad (1)$$

And the acceleration a can be calculates as

$$a[t] = v[t] - v[t-1]. \quad (2)$$

This vectorial representation contains sufficient information to generate a sound synthesis that responds to the specific properties of movement as this is being embodied. Furthermore, knowing the position of body parts we can measure distances between them and with respect to them, angles and directions.

In order to respect the computer representation of 3 dimensional space the following coordinate system will be used,



where the elevation angle φ is given by

$$\phi = cos^{-1}\left(\frac{r(y)}{\|r\|}\right); \quad (3)$$

and the azimuth angle θ by

$$\theta = tan^{-1}\left(\frac{r(x)}{r(z)}\right).$$

This idea is already exposed by ToddWinkler when he asked the following questions:

> Does human movement have constraints similar to musical instruments which might suggest something akin to idiomatic expression? Is there a distinct character to the movement of the hands? What is finger music? What is running music? What is the sound of one hand clapping? These questions may be answered by allowing the physicality of movement to impact on musical material and processes.[3]

In this way, the space described by all the possible combinations of all the body parts is restricted to the humanly possible movements and those that the device can successfully encode. This approach is now widely accessible given to the device Kinect which together with the libraries of OpenNI can produce a human model by fitting a skeleton structure that contains the joints already mentioned in an arrangement coherent to the human body. Nevertheless the system has its deficiencies and the encodable space of movements is restricted even further, more specifically, the device works better for postures in which the performer does not bend to much the torso and flexes one or two knees, or rotates. Yet, this level of fidelity is now sufficient to generate an engaging experience that gives the impression of moving in a virtual space of sound.

In short, we assume that the physical dynamics of movement encode to some extent the meaning that a gesture carries with it, both as a kinesthetic (subjective) and a kinetic (objective) experience [1]. Through this process the experience of the performer is being intervened by the resulting sound already, appealing to the simultaneous experience of proprioception of movement, and the space being activated and decoded by hearing. To put this into practice this document presents some examples of how a physics representation of movement and the moving body as a self referential system can be used to control synthesizers using gestural movements trying to preserve the encoded intentions represented by the model and emitted by the performer.

Using Position, Velocity and Acceleration

As these parameters describe the movement of each joint, their freedom of movement is restricted by their human kinetic configuration and defined linguistically by their synergic significance in the environment in which they usually move. This approach implies that the movement of the hands will bare different meanings than the movement of the feet, the knees or the torso. In this sense the expressive possibilities of the hands are greater than those offered by knees and torso for example. Also the amplitude of the movement will be greater and the changes of direction quicker.

In order to map the movement of the hands, there are various approaches that have been tried. One of them uses granular synthesis, which relies on the production of short sounds that can be defined locally through their timbre qualities, such as duration, tone, amplitude and context.

For the hands a straight forward mapping is given by the interpretation of speed as a direct proportional relation to amplitude, another can be made by changing the tone whenever the angle between two consecutive velocities passes above a given threshold, this will give us information about sudden changes of direction.

Feet move less, and their position is limited mostly to the lower halve of the scene so the mapping strategies must vary. An interesting information is the distance of a foot to the ground, this tells us if it is lifted or not, another source is the distance between feet, which combined with the previous information can describe how much effort the performer is putting into keeping balance. This last example takes us to our next approach: using the body as a reference to measure the position of joints with respect to itself.

Conclusions

As the different strategies for mapping have been implemented a double dependency was noticed while performing. The movement generates a sound that responds to it, simultaneously, the performer is being influenced by that sound to explore the virtual environment generated. This concurrent process is consistent with the idea of enactive cognition presented by Varela, Thomson and Roch, where a structural coupling takes place as a double process of exploration and behavioral adaptation to the environment [2]. In this sense we observed that the movements of the performer adapted to the restrictions of the system and to the movements that generated a satisfactory experience, restricting his/her intentions to what is possible in the virtual environment and what he/she pretends to express.

Furthermore this efficient, yet simple representation of the body in motion allows for an enriching sonic experience that maintains a correspondence between sound and the kinesthetic experience that generated it, adding yet another dimension to the proprioception of movement as an objective sonic experience.

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Special panel "Mathematics and Aesthetics in Julian Carrillo's (1875-1965) work":

"Notes on the Aesthetic Dimensions of the Sonido 13 Theory"

Keywords: scientificism, aesthetics, Sonido 13, avant-garde, microtonalism.

Abstract: In 1954, Julián Carrillo declared, regarding his microtonal theory: "The supreme aspiration to fill space with infinite sonority was the concern that moved me during the 1895 experiment, and that has continued to encourage me to fill the gaps between sound and sound: That is the aesthetic ideal of the Sonido 13 Theory!". In spite of the fact that the technical notes on such theory are many, the times in which Carrillo stops to consider his *aesthetic dimensions* are few. Moreover one may ask whether are there explicit or developed aesthetics of Sonido 13. If so, which are the traits that can be used to define such aesthetics?

This paper intends to discuss the philosophical and aesthetic thought of Carrillo by systematizing his theory. A preliminary problem that arises when doing this kind of study is the need to discern between an author's aesthetic thought and the field of aesthetic action offered by the author's own theory: the two planes are not mutually exclusive, but they must be studied as separate entities. Musicologist Gisèle Brelet explains that often creators are not clearly conscious of the aesthetics they are calling forth to express and which would allow them to remain "true to themselves", in constant renovation and exploration. In some cases happen that composers gifted with aesthetic originality do not know how to recognize this consciousness, and thus end up straying away from the promising aesthetics that other people saw as signs of their early work.

This paper stems from the fact that musical creation, like all creative human acts, is not a metaphysically inspired impulse, but intellectual and scientific work. Little does matter whether theoretical statements precede creation or are discovered along the creative process. It is essential, however, that thought reigns over creation. And yet one may ask if Sonido 13 was born from a deep reflection on the expansion of sensitive experience. Or, on the contrary, if it is a product of a modern need for a scientificist correction of technique in the arts. This research will provide the necessary tools in order to reflect upon these questions. The initial analysis will be of the *formal* (technical) imperatives and the *expressive* (aesthetic) imperatives which gave life to the creative will that urged Carrillo to develop his theory. Subsequently, lines of thought will be traced leading to the three premises upon which the edifice of Carrillo stands up. In other words, the soil upon "the aesthetics of Sonido 13" are built; namely: *Purification*, about the purity of music in its approach to physics and not to mathematics; *Enrichment*, about the quest for a sensorial infinitude; and *Simplification*, about a new musical notation that obliterates the alterations, the staff and the musical key. The study of each of these premises will bring out the positivist, scientificist and avant-garde traits that lie behind Carrillo's proposal.

The aim of this proposal is to suggest alternative ways of reflecting upon a possible aesthetics of Sonido 13 and to rescue, beyond a judgement of taste, the possibilities that this theory provides for new exploration in the field of aesthetic experience.

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"Algebraic Combinatorics on Modes"

Abstract: In the mid-1970s, Alain Louvier worked out microtonal scales called *modes of progressive transposition* and used them in many musical works. These modes have similar properties to the major modes and are related to diatonicism. Some of them were known by Ivan Wyschnegradsky and Georgy Rimsky-Korsakov, the grand-son of Nikolai. Deep scales are well known in diatonic theory, and are special cases of these modes. However, their algebraic structure is not known. Although the diatonic theories have been developed by many musicologists, such as Agmon, Balzano, Carey, Clampitt, Noll, Zweifel and others, many questions remain open. In this paper, we describe some studies on microtonality that had been published over the last century, and we review what is known and what remains to understand in this field, in both theoretical and compositional aspects.

In the first part, we study the modes called by Alain Louvier "imperfect modes". He used them in several important works as *Le clavecin non tempéré* (1973), *Canto di Natale* (1976) and *Anneaux de lumière* (1983). These modes are modes of progressive transposition. Their structural properties are highlighted in these music compositions. But Louvier used these modes only in the 24 tone equal temperament. The mathematical study would be to find a criterion to easily determine all modes of progressive transposition in any equal temperament, and in particular to determine all deep scales. The properties of these modes are very close to the properties of modes of limited transposition. In the second part, the enumeration of Messiaen modes in any equal temperament is considered. By studying microtonal modes, Louvier suggested to classify them. We present in the last section, another classification related to the plactic monoid. Along this talk, we question what, if they exist, the microtonal diatonicism and the microtonal modality could be.

Progressive Transposition Scales

In the N tone equal temperament (N-tet for short), a *scale of progressive transposition* $L = \{a_1, \ldots, a_k\}$ is a set of k pitches such that each transposition at v steps higher leads to a set $M = \{a_1 + v \bmod N, \ldots, a_k + v \bmod N\}$ such that L and M differs only by one pitch. The number v is called the *transposition index*. The scales are identified by their interval sequence, which is a word on an alphabet $\mathscr{A}$. A n-scale is a word on an alphabet of cardinality n (with n = 2 or 3). In the 12 tone system, a simple computation shows that there are only three 2-scales of 7 notes: the *major scale* 2212221 with transposition index v = 5, the *pseudo whole scale* 1122222 with transposition index v = 2, and the *chromatic scale* 1111116 with v = 1: It is remarkable that the only tame scale of 7 notes is the major scale. In the 24 tone system, the number of scales of progressive transposition with 2 or 3 letters ranges from 1 to 180. The scale of 7 notes 4334343 was used by Alain Louvier in *Aria, récit et carillon* (*Le clavecin non tempéré no. 1*). It is like a major scale (C, D, E↓, F, G, A↓, B↓) with modal degrees (E, A and B) lower by a quarter-tone.

Deep Scales

A *deep scale* is a scale containing each interval class a unique number of times. It follows that the deep scale property ensures there is a different number of common tones associated with each transposition level with one exception. By definition, a deep scale is trivial if its transposition index is 1 or 2. The only non trivial 2-deep scales maximally even are the scales $12^{n-1}\,12^{n}$ for $N = 4n$ and $n \geq 1$ is an integer (1^n means 1 repeated n times). In the 12-tet, it corresponds to the major scale, and in the 24-tet, it is what Wyschnegradsky called *diatonicised chromatism*. These scales are some archetypes of generalized diatonic scales. In addition, we will show that to determine if a scale M is deep or is a progressive transposition scale in the N-tet, it is sufficient to look at the subwords of the interval sequence.

Microtonal Diatonic Scales

Usually, a diatonic scale is a well-formed scale maximally even. But it has been shown that if A is maximally even, then $N = 2(k-1)$ and $N \equiv 0 \bmod 4$. It follows that the definition of diatonicity is not suited for all chromatic universes. That is why several theories emerged. Eytan Agmon found two kinds of diatonic scales depending on the parity of N. In his theory, the diatonic scales are $2^{(N-1)/2}1$ if N is odd, and $2^{(N/2-3)}12^{(N/2-4)}1$ if N is even. All these scales are maximally even and well-formed. In the 24-tet, the *diatonicized chromatic scale* is a scale of 13 notes, constructed by two connected heptachords. Wyschnegrasky used this scale in his *24 Preludes op. 22* and *Premier fragment symphonique, op. 23*. In this section, we give some others definitions of generalized diatonic scales. For example, if $N = 13$, the scale 22122121 is different from Agmon's diatonic scale, but is the same as the one of Erv Wilson (as we can see on his keyboard plan). It is another way to consider diatonicism by introducing deep scales. For an integer $p \geq 3$, we describe the *p-olic diatonic scales* which are well-formed and deep scales over the alphabet $\{1, p\}$.

Microtonal Modes of Limited Transposition

This section deals with the enumeration of modes of limited transposition. Modes of limited transposition are well-known in the 12-tet since Olivier Messiaen has used them in many compositions. But it is a rather difficult question to give a way to construct these modes and to enumerate them in a given N-tet. In 1982, we first studied modes of limited transposition in quarter-tone system and found 381 modes. With F. Ballon, we give a complete answer and found the formulas to enumerate them.

These modes are not just theoretical speculation. In the 24-tet, Georgy Rimsky-Korsakov used the scale 33333333. More recently, Alain Louvier wrote *Prelude et Fugue no. 2* (1978) (*Le clavecin non tempéré no. 2*) in the 18 tone system. In this work, Louvier used a mode of limited transposition of interval sequence: 111311131113. In the same way, in the 24-tet, he used in *Prelude et Fugue no. 3* (1973) (*Le clavecin non tempéré no. 3*) the mode of limited transposition 111117111117.

Plactic Modes Classification

As modes are relatively large, the goal of this section is to classify them. There are many ways do to so. Here we try to classify them by using *plactic relations*. Modes are identify by their interval structures, or more abstractly by letters a,b,c, etc. The plactic monoid over some totally order alphabet A= $\{a,b,c,\ldots\}$ with $a<b<c<\ldots$ is the monoid whose generators are the letters of the alphabet verifying the Knuth congruence relations

$$\begin{cases} bca \equiv bac \text{ whenever } a < b \leq c \\ abc \equiv cab \text{ whenever } a \leq b < c \end{cases}$$

In the 12-tet, the 14-modes class of some heptatonic modes is composed by some church modes and karnatic modes. In the 24-tet, the class of 14 modes (with $a = 1$, $b = 2$) in the 12-tet remains the same class in the 24-tet (with $a = 2$, $b = 4$). The dual class (reverse each word and change the name of the letters) of 14 modes has two new implementations ($a = 2$, $b = 7$ and $a = 3$, $b = 4$). The heptatonic mode 4334343 used by Alain Louvier in *Le clavecin non tempéré* belongs to this class.

Conclusion

Since the use of microtones is nowadays a standard in contemporary music, some composers like Alain Bancquart, Warren Burt, Pascale Criton, Dean Drummond, Georg-Friedrich Haas, Ben Johnston, Bernhard Lang, Michaël Levinas, Joe Maneri, Jean-Étienne Marie, Laurent Martin, Bruce Mather, Pauline Oliveros, Gérard Pape, François Paris, Enno Poppe, Alberto Posadas, Henri Pousseur, Horatiu Radulescu, Johnny Reinhard, Franz Richter Herf, Marc Sabat, Ezra Sims, Martin Smolka, Manfred Stahnke, Karlheinz Stockhausen, James Tenney, Lasse Thoressen, Toby Twining, Samuel Vriezen, Julia Werntz and many others, have shown different approaches in their use of microtones. Today, new microtonal investigations require further studies in microtonality. From the first paper of Georgy Rimsky-Korsakov to the one of Alain Louvier in 1997, and to some more recent papers, the investigation of microtonal modes is a great way for understanding diatonicity.

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Johnson, Tom & Samuel Vrizen
Composers (from Paris & Amsterdam)

"Informal talk between Tom Johnson and Samuel Vriezen"

Keywords: Music and mathematics, experimental/constructivist music, combinatorics, block diagrams, *Looking at Numbers*.

Abstract: Tom Johnson has been engaged translating mathematical patterns into music for quite some time, having built a considerable musical oeuvre tackling the issues involved from many different perspectives. Not being able to attend this Congress personally, Johnson agreed in participating live in a video conference here with his longtime collaborator and colleague, pianist/composer/writer Samuel Vriezen, who will also be playing a program that includes pieces by both of them.

They will address common interests, the relations between music, mathematics and composition, the pieces to be played and recent work. Also on the agenda is *Looking at Numbers*, a recent book on visualization and sonification of numeric patterns written by Johnson with Franck Jedrzejewski. The book contains a collection of visual approaches to mathematical problems (mainly in combinatorics), some of which arise from specifically musical questions. That is the case of block designs, which happen to be a meeting point in the relation between the two guest speakers.

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Special panel "Mathematics and Aesthetics in Julian Carrillo's (1875-1965) work":

"Microtonality and the Music of Julian Carrillo from a Xenharmonic Perspective"

Keywords: Microtonal, just intonation, equal divisions of the octave, EDOs, regular temperament, meantone, linear temperament, metatonal, xenharmonic.

Abstract: Today, microtonality has both, a theory that greatly exceeds common practice, and a common practice of music that often enters microtonal territory, but is not described outside the concept of 12edo. Microtonality is also a strange term for music which can have basis in tunings that contain larger intervals than common practice, for example 5edo which has 240c steps, or 7edo which contains 171c steps. The multiples of these tunings (10, 15, 14, 21 etc. edos) also contain these same larger intervals. *Metatonal music* or *xenharmonic music* have been proposed as terms to describe this music. I believe that this music will eventually not be thought of as a subcategory of 12edo, but the other way around, as this music offers description that transcends cultural, historical, and technological bounds.

Summary of JI

Just intonation (JI) is a tuning in which the relationship between frequencies of notes in a scale can be described by ratios of whole numbers. Equal division of the octave tunings (EDOs) are described by multiples of the roots of 2 equal to the division of the octave (ex. ratio between each consecutive interval of 12edo is equal to the 12th root of 2 : 1). The reason for the use of EDOs over JI can be illustrated in the fact that when stacking the frequency ratio 3:2 (P5), a ratio of 81:64 is created which is a "major third", but a major third tuned to the ratio 5:4 is more likely to occur when someone tunes the M3 to the tonic by ear, as opposed to tuning consecutive 5^{ths} (3/2s) by ear. The 19edo for example tunes the M3 very close to 5:4 (closer than 12edo) but at the expense of the P5 being flatter than 3:2 in both JI and 12edo (however, the M3 in 12edo is further to JI than the P5 in 19edo is to JI).

Summary of 1–31 edos

The equal divisions of the octave 31 and lower have been the centre of my interest in microtonal music for the past few years. Just intonation was the focus prior. 1edo is only a single note. 2edo is the "tritone" of 12edo, aka the frequency ratio of sqrt(2). Tritone becomes a misnomer here as it is no longer 3 "tones" of 12edo. 3edo is the augmented triad that exists in

12edo. Each interval of 3edo is equal to the frequency ratio of 3rt(2). Furthermore, each interval of *n*-edo is equal to the frequency ratio of (*n*)rt(2). 4edo is the diminished 7th that occurs in 12edo. 5edo in an equally spaced pentatonic scale in which each interval is 9c flat of 8/7 (and plays a large role in 15edo, which can be thought of as three interlocking 5edos). 6edo is the whole tone scale that exists in 12edo. 7edo sounds not too distant to a 12edo major scale even though the M3, P4 and P5 arguabley do not exist. 8edo is two interlocking diminished 7th triads (as found in 12edo) which are dissonant to each other. 9edo is three interlocking augmented triads (as found in 12edo), and approximates the septimal minor third (7/6) almost exactly.

My research of equal divisions of the octave has been centred around their similarities, differences, and their unique properties, as opposed to a search for optimization. For example, 14edo has two interlocking 7edo note scales which both loosely approximate the major diatonic scale. 15edo has three interlocking 5edo scales which are similar to the minor pentatonic scale (but tuned more to the 7th harmonic instead of the inverse of the 5th). 16edo contains an "anti-major scale" with the pattern of ssLsssL or 2232223 and has a P5 tuned so flat that four of them stacked will approximate a minor third instead of a major third. 19edo works similarly to 12edo in many ways. 19edo contains a major triad that sounds arguably better than in 12 (with the M3 far closer to 5/4 than the sharp M3 in 12edo, but at the expense of a flatter P5, which is still closer to 3/2 than the M3 of 12edo is to 5/4). 19edo also contains a diatonic scale with step sizes of 3323332 which works equivalently to the 12edo major scale pattern of TTsTTTs.

15 edo & Porcupine Temperament

15 equal divisions of the octave contains "Porcupine Temperament". Porcupine is a linear temperament which tempers out the comma 250/243, which is called the Porcupine Comma or the Maximal Diesis which is defined as the difference between: three 10/9s and 4/3, three 6/5s and 16/9, and 81/80 and 25/24. In this temperament, 2 perfect fourths are equal to 3 minor thirds. This can be seen in 15edo where the P4 is a 6 step interval and the m3 is a 4 step interval. This can also be thought of as breaking the P4 into 3 equal parts and the m3 being broken into 2 equal parts. 22edo also contains Porcupine Temperament where the P4 is a 9 step interval, and the m3 is a 6 step interval.

15edo creates great 10 note major like scales with the tone pattern of 2121212121. This can also be thought of as two 5edo scales 80c apart. With 5edo as a tonal basis, all of the other notes in 15edo will work well as leading tones. This creates an interesting psychological effect while playing in 15edo where you feel both like you have more and less notes than 12edo. The usefulness of small # edos in practice is illuminated by the paradigm shift in 15 where a "major scale" can be thought of as 2 interlocking pentatonic (equally spaced) scales and can contain 10 notes.

A guitar fretted to 15edo with strings tuned to a JI major or minor triad (like open G or D tuning on a 12edo guitar) creates a system that hybrids the symmetries and 7-limit nature of 15edo while also containing a movable 5-limit JI triad. Tuning flexibility on instruments such as the guitar unlock a huge amount of sonics resources of this nature. Another example is in 16edo, where if the 675c P5s are tuned sharp to the 700c of 12edo on some strings, the instrument becomes a subset of 48edo (one of Carrillo's tunings) and captures some of its best qualities (including accurate 5-limit harmony) without the practical struggle of such a large # edo tuning.

19 edo

19 equal divisions of the octave is a great tuning for 5-limit triadic harmony, meantone, and for approximating the same 7 note diatonic scale that 12edo does. It approximates this scale with the same step pattern too, TTsTTTS or LLsLLLs, except instead of 2212221, it is 3323332. Another way to look at 19edo from the perspective of 12edo is to take the black key enharmonics from 12edo and separate the sharps from the flats; C# and *D*♭ are now different notes. In addition, B# will no longer be C, nor *C*♭ B, but B# will = *C*♭ and be an independent note, likewise with E#/*F*♭. These 7 additional senses allow 19edo to function very similarly to 12edo, and even to use an identical notation system to 12edo.
Like 12edo, 19edo has a strong basis in Meantone Temperament. Meantone temperament is the predecessor to the concept of 12 equal temperament. In meantone, four 3/2s = 5/1. This is equivalent to saying that the M3 is tuned by stacking four P5s. This was the method of tuning in Bach's time and created various meantone tunings, well-temperaments, and eventually paved way for 12edo. Meantone equates 9/8 and 10/9 (tempers out 81/80, the syntonic comma) and hence creates a M3 comprised of two equal tones. Other EDOs that contain meantone are 19, 31, 43, 50, 55, and 81. The 10/9 "minor tone" is also the difference between the JI P5 and M6 and the tempering of it therefore plays a large role in the natural minor scale in 12edo being a "mode" of the major scale.

Carrillo's EDOs from a xenharmonic perspective

24edo contains all of the harmony of 12edo as it is a multiply of it, yet, due to where these 12 additional notes are located, it does not expand on the 5 or 7 limit harmony that exists in 12. It does, however, introduce the 11th limit. This may be deterring factor for those whose first escapade into microtonal music from 12edo is 24edo. 24edo is often used to represent tuning in Arabic music. This is, however, a very limited relation, as Arabic music is primarily melodic in nature and is very aural in tradition, with few instruments being tuned to the precision of a piano or a guitar. The very nature of the instruments of Middle Eastern music show a desire for pitch flexibility.

36edo not only divides each semi-tone in 12 edo into 3 parts, but due to the highly composite nature of the number, also divides each whole tone in 12 into 3 parts. 36edo, like 24edo, also offers no improvement on the 5-limit harmony of 12edo. However, it does improve the approximations to 7 limit harmony. 36edo also offers a lot of resources for atonal music, and can be thought of 6 interlocking whole tone scales (the 12edo whole tone scale is exactly 6edo).

48edo can be conceived both as dividing each 12edo interval into 4 parts, and as dividing each interval of 16edo into 3 parts. "Something close to 48edo is what you get if you cross 16edo with pure fifths, for instance, on a 16-tone guitar. The presence of 12/11 in 16edo allows a string offset of 11/8 to also work for producing perfect fifths" (quoted from Xenharmonic wiki).

72edo is used as a theoretical construct for tuning Turkish music and is good EDO for *miracle temperament*. Miracle temperament divides 3/2 into 6 equal parts. The generating interval in miracle is called a "secor" and is between 16/15 and 15/14 (116.7c). 225/224, called the Septimal Kleisma, the difference between 16/15 and 15/14 is hence is tempered out. Two secors make an 8/7 interval, the septimal whole tone. Two 8/7s or 6 decors make up a 3/2, therefore 1029/1024 is also tempered out (the difference between two 8/7s and 3/2). The neutral third of 11:9 is also approximated by 3 secors. 72edo works well for miracle temperament because the secor is approximately 7 steps of 72edo. 72edo is a very accurate tuning for 3, 5, 7 and 11 limit harmony, yet is incredibly large. The secor is also approximated in 31edo, 41edo, 72edo, and Harry Partch's 43 tone JI scale.

96edo is the largest # EDO used by Carrillo. Like all EDOs in this range, 96edo offers quite accurate approximations of many common JI intervals. The varying degrees of accuracy and usefulness in this region (even above 36edo) are outside my field of study at the moment.

Conclusion

Small equal divisions of the octave offer a plethora of interesting and useful properties for composers and musicians and theorists alike, from tunings such as 19edo or 31edo which can be thought of as expansions of 12edo and meantone, but with a completely different tone set, to tunings such as 14/15/16edo which offer strange and beautiful new patterns and symmetries, to the tunings of Julian Carrillo which explicitly expand 12edo.

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Notes: Here *metatonal music* refers to a guitar maker from Florida who specialized in making and modify guitars to equal divisions of the octave between 12-46edo. *Xenharmonic music* refers to the internet community "the Xenharmonic Alliance II".

Lach, Juan Sebastián
Conservatorio de las Rosas (Morelia)

Special panel "Mathematics and Aesthetics in Julian Carrillo's (1875-1965) work":

"Compositional research into the logics of pitch-distance and the timbral facet of harmony in Julián Carrillo's *Leyes de Metamorfosis Musicales* (Laws of Musical Metamorphoses)"

Keywords: Harmonic theory, harmonic duality, early microtonal pioneers.

Abstract: Pitch materials in music can be understood as having two components, pitch-distance and proportionality. Both aspects are intertwined and difficult to separate, but can be distinguished by studying their properties. The distance aspect is related to spectrum and inhabits the continuous logarithmic space of pitch glissandi; its main qualitative effect is sensory consonance and dissonance. The proportional aspect is related to intervallic properties linked through numbers and can be represented in a multidimensional lattice of discrete points that lies within the pitch continuum; its main quality is harmonicity, which does not always coincide with sensory consonance. Following James Tenney [2], who theorizes musical form as consisting of morphology (contour, continuous variations of musical variables) and structure (relations between parts and wholes) at different temporal levels, we could also say that the proportional aspect of harmony relates to structure and the timbral one to morphology.

Carrillo's research in *Leyes de Metamorfosis Musicales* (1949 [1927]) can be read from the viewpoint of this formal and harmonic duality. Even if he did not know or intend something resembling this theoretical perspective, it can nevertheless reveal how his involvement with the properties of the pitch materials unleashed by his divisions of the octave was not arbitrary. His proposal of scaling musical morphologies in pitch and time can be seen as a consequence of his search for an intrinsic logic to the types of intervals and tuning systems he was using. Given that his approach took place along the pitch-distance axis of the harmonic dichotomy, his findings reveal some important attributes of this facet, anticipating, for example, some features of Iannis Xenakis' work [5]. His method can also be compared to the research of his younger contemporary, Augusto

Novaro, who instead of a timbral perspective, approaches intervals proportionally before searching for the equal divisions of the octave which best approximate them. This comparison can shed some light on the current landscape of microtonal composition.

Despite the naiveté of some of Carrillo's writing and his sometimes dogmatic statements, and beyond his complex, difficult personality and obscuring biographical anecdotes, we want to show that his ideas still carry an undeveloped potential that can be expanded and adapted (for example, by taking their perceptual and aesthetic effects into account) to today's compositional situation.

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"Proportion, Perception, Speculation: Relationship between Numbers and Music in the Construction of a Contemporary Pythagoreanism"

Keywords: Pythagoreanism, musical qualities of numbers, music and mathematics, continuous-discrete dichotomy, sonification.

Abstract: Music, as a practice, a form of knowledge and and art form, relates in many ways to mathematics. In particular, there is a link between whole numbers and intervallic pitch perception that stems from Pythagorean discoveries and has developed through time into a collection of compositional harmonic resources that widens the space of possibilities afforded by the tonality/atonality dilemma. Our research attempts to delve into qualitative musical-numeric connections in order to assess some mathematics that could be associated with musical structures at different temporal scales. The approach is speculative in nature, in the sense that it is not only related to empirical or music-theoretical research but has an experimental dimension, extrapolating ideas in order to suggest points of departure in composition.

The mathematical opposition between the continuous and the discrete encompasses many unresolved aporias throughout Western thought. It also surfaces in music at several levels. We will propose a map that traces musical structures at various time scales by describing types of musical phenomena at each level and in relation to abstract mathematical objects at both sides of the discrete/continuous polarity. These mathematical structures can be used analytically, but their main purpose is to synthesize musical forms, working as possible logics that are immanent to their materials (i.e., being also part of the playing field). Abstraction is understood in the sense of physical (or acoustic) 'concentration' rather than as a higherorder representation; it is induced through a statistics of perceptual thresholds (perception broadly understood as a filtering process and multi sensory space-time synthesis) and allows us to think and put forward strategies to navigate each temporal realm.

Perceptual temporal integration determines the *micro* scale, where harmonic arithmetic and timbral spectrality play a role; as differentiation gives way to plurality and multiplicity of sound phenomena in the *meso* scale, it is combinatorics that can serve as a guiding thread; at the *macro* scale of form, where memory as intensive presence has an important effect, maybe a more diagrammatic, non-linear approach can be feasible, one that can encompass different aesthetic models of experience. We could even conceive a bigger scope, having to do with awareness beyond a conventional musical performance, as in the case of very long pieces or installations and related to the 'atmospheric' aspect of sound experience.

Each temporality has its own consistency and logic and the limits between scales are not fixed and can also be considered intermediary spaces of their own. To illustrate these approaches, some relations between mathematics and music in contemporary music will be discussed with a view to suggesting numbers and structures that are interesting for sonification.

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Welcome lecture (2):

"On the Relationship Between Music and Mathematics"

Keywords: Mathematical Music Theory, mathematical structures.

Abstract: In these words of mathematical and musical welcome, a short view of the last three decades of this field mathematically speaking in Mexico will be addressed. We will give a brief description of how it began and what has being done. Also we will comment briefly on the relation between mathematics and music.

References: [1] Agustín-Aquino, O.A. & E. Lluis-Puebla (Eds.) *Memoirs of the Fouth International Seminar on Mathematical Music Theory*. Publicaciones Electrónicas de la Sociedad Matemática Mexicana. Serie Memorias. Vol. 4. 2012. [2] Lluis-Puebla, E., G. Mazzola & T. Noll (Eds.) *Perspectives in Mathematical and Computational Music Theory*. epOs. Osnabrück. 2004. [3] Lluis-Puebla, E. *Música, Matemática y Concertismo*. Publicaciones Electrónicas del Instituto Mexicano de Ciencias y Humanidades. 2011. Mazzola, G. *The Topos of Music*, Birkhäuser. 2002.

Lobato-Cardoso, Jaime & Juan Antonio Martínez-Rojas
ENM – UNAM (Mexico City) & Departamento de Teoría de la Señal y Comunicaciones, Universidad de Alcalá (Madrid)

"*Topos Echóchromas Hórou*: the Place of Timbre of Space"

Abstract: Recent researches on embodied cognition have developed major advances in the study of human echolocation, not only by developing theoretical models, but protocols for teaching it as well. This sensory quality that humans possess allows us to obtain data on dimensionality of space, and expanding our capacities to interact with the surrounding environment.

From a Spatial Composition Method proposed by co-author Lobato-Cardoso, were related: binaurality, echolocation and evanescent perception (new technique discovered and developed by co-author Martínez-Rojas). This paper proposes making a comparative analysis of different types of geometry and acoustic phenomena which allow us to perceive a three-dimensional space through the ear.

The main goal elaborating this first comparison is to set a precedent for the development of alternative methods of teaching geometry and topology, as well as build a conceptual bridge between Spatial Composition Method and traditional systems, through the geometrical description of timbre.

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"Models and Algorithms for Music Generated by Physiological Processes"

Abstract: Physiological processes give rise to a wide variety of signals. These signals in turn can be detected by changes in pressure, temperature, electrical potential and so on. When measured and converted with the appropriate transducer, they constitute the raw material which algorithms and models may translate into sound. In this lecture we explore some specific models and algorithms in different physiological contexts.

Loy, Gareth
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"Music, Expectation, and Information Theory"

Keywords: emotion in music, entropy, expectancy, surprisal, uncertainty.

Abstract: Music is successful in a Darwinian sense if listeners attend to it, which they do if they find it entertaining. Listeners are entertained by the interplay between their expectations and the musical facts that they experience. Expectation is anticipatory belief, which ranges from certainty to uncertainty. The interplay of certainty and uncertainty in the listener's expectations relates to the affects of satisfaction and surprise. Music requires a degree of structural ambiguity—enough, but not too much—to balance listener's expectations, and thereby gain and maintain interest. The concept of surprisal from information theory suggests a computational perspective on the psychology of musical expectation that may help form the basis of a theory of musical attention.

Though the examples in this presentation draw from Western classical music, the hope is that researchers will be encouraged to investigate how the ideas presented can be applied to music of other cultures and eras, leading to a mathematically grounded universal theory of music.

References: [1] Aristoxenus (c. 300 BC). *The Harmonic elements*. In *Source readings in music history: Antiquity and the middle ages* (ed. O. Strunk, 1950), pp. 27–31. W.W. Norton & Co., New York (p. 31). [2] Cage, J. (1958). *Fontana Mix*. [3] Freyd, J. J. (1987). "Dynamic mental representations". *Psychological Review*, 94 (4): 427–438. [4] Haydn, J. (1791). *Symphony no. 94* ("Surprise", 2nd movement). [5] Langer, S. (1942). *Philosophy in a New Key: A Study in the Symbolism of Reason, Rite and Art*. Harvard University Press. [6] Meyer, L. (1956). *Emotion and Meaning in Music*. University of Chicago Press. [7] Nyquist, H. (1928). "Certain topics in telegraph transmission theory." Trans. AIEE, vol. 47, pp. 617–644, Apr. 1928. [8] Schoenberg, A. (1923). *Suite for Piano Op 25 – Part II*. [9] Shannon, C. & W. Weaver (1949). *A Mathematical Model of Communication*. University of Illinois Press, Urbana, IL. [10] Shannon, C. (1948). "A mathematical theory of communication". *Bell System Technical Journal* 27 (July and October): pp. 379–423, 623–656.

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"Project DRUMMATH: Rhythms that Build Meaning in Mathematical Concepts for the Visually Impaired"

Keywords: Drummath, special needs of visually impaired students, Mathematics Education, rhythms.

Abstract: The Project DRUMMATH is a research on how visually impaired children learn mathematics and on how sounds can be used as sources for building images. It focuses on the development of pedagogical activities designed to help students build meaning around mathematical concepts at early age. The activities developed on Project DRUMMATH demand students to count and perform movements, clapping their hands and feet, on top of musical backgrounds (like sounds and rhythms). The lecture will focus on two different moments of the history of Project DRUMMATH's development. The first one when tools from number theory (such as modular congruences equations of higher degrees) were used to describe rhythms, melodies and harmonies. The second moment regards how the mathematical description of rhythms turned out to be the very basis of the development of pedagogical activities toward the visually impaired. Several activities that were developed on a special Brazilian school for the visually impaired children will be shown and also executed by the participants. The lecture will gather pure aspects of number theory, learning theories that concern special needs.

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PLENARY LECTURE:

"Gestural Dynamics in Modulation—A Musical String Theory"

Keywords: hypergesture, Modulation Theory, Stokes theorem, Escher theorem, String Theory.

Abstract: In a recent book [1, Ch. 9:10], we have presented a restatement of basic theorems of mathematical counterpoint theory in terms of the mathematical theory of musical gestures [2,3]. The present paper aims at a hypergestural discussion of classical mathematical modulation theory [4,5,6]. Following that approach, it can be proved that tonal modulation as described by Arnold Schoenberg can be modeled using symmetries between scales underlying the involved tonalities. For example, to modulate from C-major to F-major, Schoenberg proposes the three modulation degrees II_F, IV_F, VII_F. These degrees also come out from the mathematical model, where the C scale is mapped to the F scale using the inversion symmetry $S=T^9.-1$ between e and f. The mathematical model yields exactly Schoenberg's modulation degrees in all cases where he describes a direct modulation, namely for fourth and fifth circle distances 1,2,3,4.

The present approach is based on the idea that degrees in the start tonality are interpreted as being gestures that move to degrees (qua gestures) of the target tonality be means of hypergestures. This means that the symmetries relating tonalities in the classical setup are replaced by hypergestures that connect gesturally interpreted degrees. Although we are still in search for a theory that might generate natural "minimal action" hypergestures in the sense of Hamilton's variational principle in mechanics [7], the present hypergestural model solves the problem. The classical modulation model was driven by the idea of elementary fermion particles in physics, interacting via bosons that materialize interaction forces. The hypergestural restatement of this model views symmetry-corresponding degrees as being musical fermions being connected via a boson hypergesture. We also interpret this hypergestural approach as a complete parallel to S-duality in physical string theory [8].

The general procedure is as follows: We first model gestures and hypergestures in the real plane $\mathbb{R}^2$, where the usual pitch class set is embedded on a circle. We then look at triadic degrees of pitch class points, which are represented as gestures of lines connecting these points. Next, we construct vector fields on $\mathbb{R}^2$ whose integral curves give rise to hypergesture curves that deform the gestural degrees. This field is the Lie product [X,Y] of two fields, X and Y, that represent the inner symmetries of the start and target tonalities of our moduation. Figure 1, left, shows this Lie product, together with the pitch class circle. To the right of Figure 1 we show the three integral curves of [X,Y] that comprise all twelve pitch classes. The deeper reason why only three integral curves suffice to contain all twelve pitch class points is still not understood, but it is a fact. The field [X,Y], together its integral curves share a reflection symmetry which on the pitch classes coincides with the symmetry $U_{e/f}$ that maps the start tonality C to the target tonality F.

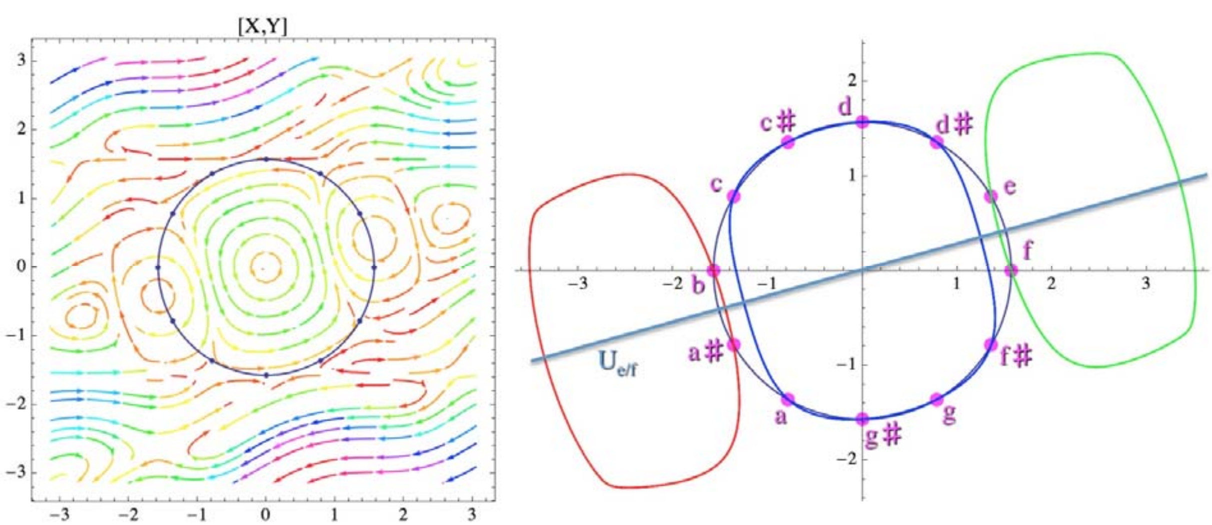

Figure 1. Left: The vector field [X,Y], together with the circle containing the twelve pitch classes. Right: The three integral curves of [X,Y] that comprise the twelve pitch classes, together with the field symmetry $U_{e/f}$.

Then we discuss cadences of such triadic degrees and their behavior under hypergestural deformation. We prove that for a specific choice of such vector fields, the inversion symmetries used in the classical model map pitch classes x into pitch classes living in the integral curve of x. Next we consider the trajectories of the curves of the Escher-Theorem-inverted perspective and calculate energy integrals of such curves. Under the condition of non-vanishing energy, we can then exhibit the admitted degrees. These integrals refer to Stokes' theorem for hypergestures. We conclude this paper with a statement of this theorem and a sketch of a proof.

References: [1] Agustín-Aquino, O.A., J. Junod, G. Mazzola: *Computational Counterpoint Worlds*. Springer, Heidelberg 2014. [2] Mazzola, G. & M. Andreatta: "Diagrams, Gestures and Formulae in Music". *Journal for Mathematics and Music*, 1 (1): 23-46, 2007. [3] Mazzola, G. "Categorical Gestures, the Diamond Conjecture, Lewin's Question, and the Hammerklavier Sonata". *Journal of Mathematics and Music*, 3 (1): 31–58, 2009. [4] Mazzola, G.: *Gruppen und Kategorien in der Musik*. Heldermann, Berlin 1985. [5] Mazzola, G.: *Geometrie der Töne*. Birkhäuser, Basel 1990. [6] Mazzola, G., *et al.*: *The Topos of Music*. Birkhäuser, Basel 2002. [7] Abraham, R.: *Foundations of Mechanics*. Benjamin, New York, 1967. [8] Hashimoto, K.: *D-Brane*. Springer, Heidelberg 2012.

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"A Computational Tool for Image Sound Synthesizing"

Abstract: A sound synthesizer is an electronic device capable of producing a wide range of sounds by the generation of wave signals designed to imitate other instruments or generate new timbres employing different synthesis techniques [1]. Synthesizers may consist of one or more modules such as oscillators which generate with different waveforms (i.e., sine, saw, square), audio filters, envelope controllers, delay controllers, sampling recorders, etc. The creation of a new sound with a synthesizer may be performed by the subjective adjustment of these parameters until a pleasant sound is achieved. Obviously, in order to create a new sound with specific properties or imitate a specific sound, it is necessary to have some knowledge of the principles of sound and the impact of each of these parameters in the final sound of the synthesized instrument.

A spectrogram is a tool that allows the visualization of the frequencies that generate a sound, and it is very useful for different applications in engineering and science including speech processing and identification and characterization of sounds [2]. A spectrogram is a image or array of numeric values that represent the intensity for an specific set of frequencies over time. It may be generated from an analogous sound signal by the use of pass-band filter banks tuned at different frequencies. For digital sound signals, the spectrogram can be computed using the Short Time Fourier Transforms (STFT).

Given a sound signal S digitized with a sampling frequency Fs, the generation of an spectrogram by means of the STFFT consist of defining sections or chunks of a determined number of samples c and with a fixed percentage of overlapping O between the chunks. Each of the chunks is filtered employing a filter window (e.g., Hamming, Hanning, and Blackman). The Fourier transform of each filtered chunk is computed using the Fast Fourier Transform algorithm to produce a vector of complex values on which each of its element is associated with an specific frequency. The magnitude of the complex values at every element of the vector corresponds to the intensity of the corresponding frequency that generates the wave of the digital sound in the period of time determined by the signal chunk. The magnitude values are mapped to a discrete value in the interval [0255] and treated as a gray scale image or as a colored image using a color map.

A sound wave may be reconstructed from its spectrogram. This brings the possibility of generating new sounds from defined patterns in an image. Moreover, the edition of a specific sound may be possible by the modification of the principal frequency components that defines it. However, currently this procedure must be performed manually on an image edition tool due to the lack of computational tools specifically designed for this purpose.

In this work we present computational methods for the reconstruction of sounds from spectrograms images as a linear combination of oscillating functions with specific frequencies for which the weights are determined by the intensities of the spectrogram images. We introduce different tools to help the user in the creation of a new sound which considers the different properties of the pleasant sounds such as the use of harmonics at different multiples of the fundamental frequency. Additionally, we propose a method for the identification and segmentation of the principal frequency components of an spectrogram image by the use of image segmentation methods commonly employed in computer vision which allows the creation of a new sound based on existing sounds.

References: [1] Vail, M. *Vintage Synthesizers: Groundbreaking Instruments and Pioneering Designers of Electronic Music Synthesizers*. Backbeat Books, 2000. [2] Flanagan, J.L. *Speech Analysis, Synthesis and Perception*. Springer-Verlag, 1972.

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"Manuel M. Ponce's piano *Sonata No. 2* (1916): An Analysis Using Signature Transformations"

Keywords: Signature transformations, Neo-Riemannian transformations, Manuel M. Ponce, diatonicism, modality.

Abstract: In the present work an analysis is made of several passages from Manuel M. Ponce's (1882–1948) *Sonata No. 2* for piano, employing Julian Hook's theoretical development of *signature transformations*. Ponce's *Sonata no. 2* has clearly a nationalist character. The two themes of the first movement are borrowed from two folksongs, *El sombrero ancho* and *Las mañanitas*, and the first theme of the second movement is based on the traditional Mexican "son", *Pica, pica, perico*. The date of this composition, 1916, falls in what is still considered Ponce's "romantic period", opposed to his "modern style" of later years [see: 3]. Nevertheless, when studying this piece, one finds a style that is far from the formal characterization of the period we know as Romanticism; the first movement of the *Sonata* is full of non-traditional chord progressions, of dissonance, hints of modality, and the influence of the impressionism of Debussy—so admired by Ponce.

Inside the neo-Riemannian focus there have arisen several forms of carrying out theoretical analysis of a score by means of mathematical transformation groups. There is an undeniable coincidence among these forms but, at the same time, each one offers unique aspects that privilege the specificities of the piece itself and the needs of the analyst. In this work we will make use of *signature transformations*, fruit of the theoretical development of Julian Hook [see: 4]. Signature transformations act on the set of *fixed diatonic forms* [4 : 140–142]. Fixed diatonic forms are equivalence classes of fragments of diatonic music, with a key signature and a clef. These fragments are in the same equivalence class if their pitch-class content is the same (modulo 12), and if they determine the same diatonic collection up to enharmonic equivalence.

We will use the notation s_n, $n \in \mathbb{N}$, for the number n of sharps that are added and s_{-n} for the number $-n$ of flats that are added. The operation of adding sharps (or subtracting flats) is "positive", and the operation of subtracting sharps (or adding flats) is "negative". For example, in figure 1, s_{-6} reduces the key signature by 4 sharps and then we continue to count negatively by adding flats:

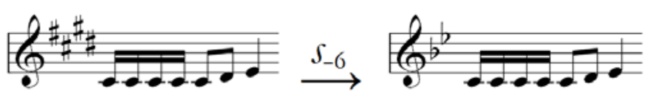

Fig. 1 Signature Transformation.

The signature transformations form a cyclic group of 84 elements (they pass through the twelve pitches of the chromatic scale and the seven diatonic modes) generated by s_1, although it is not expected that 84 sharps would be added to a key signature! Indeed, even though the signature transformations form a cyclic group, the group of signature transformations can be looked at as compositions with *Schritts*, given that the s_n and s_{-n} can be reached through compositions with T_n and t_n, the chromatic and diatonic transposition operators respectively.

If we add seven sharps to a key signature we will transpose the diatonic collection a semitone (for example, from *C* major to *C#* major). Therefore, s_7 operates in the same way as T_1 and, analogously, s_{-7} acts as T_{11}. Hence the validity of compositions such as $T_7s_2 = (T_1)^7s_2 = (s_7)^7s_2 = s_{51}$ and the perspective of composition with *Schritts*.

As an example of how signature transformations work, we will trace the changes from measures 227 and 228 to measures 231 and 232 of the first movement of Ponce's *Sonata no. 2*. Measures 227 and 228 are in *E* Aeolian, which only has one sharp. This is reflected in Figure 3, but in the original piece the key signature has four sharps, as can be seen in Figure 2. Measures 231 and 232 are in *G#* Aeolian, which has five sharps. To travel from *E* Aeolian to *G#* Aeolian we must add four sharps by the application s_4 (which places us in *E* Lydian) and then transpose diatonically by two tones. We emphasize that, as the composition is commutative, it could have also been carried out in the inverted order (although there are examples in which it is not possible musically to carry out some s_n in particular, due to the diatonic context). Hence, the signature transformation is $t_2s_4 = (t_1)^2s_4 = (s_{12})^2s_4 = s_{28} = s_4t_2$. Of course, we can look at this transformation as simply $T_4 = (T_1)^4 = (s_7)^4 = s_{28}$.

Signature transformations, once they are understood and practiced, are a friendly and powerful tool for surfing among the 84 possibilities of the seven diatonic modes and twelve possible tonics. This mathematical formulation is ideal for analysis and reflects the mastery of its creator in the use of the algebra that underlies it; it also makes patent the necessity of this type of analysis, in which the shifts in these rigorously defined, but at the same time flexible, diatonic collections (*diatonic forms*) can be traced in a precise manner. They show the ongoing validity of the philosophy of multiple perspectives, that gives rise to new mathematical constructions which are capable of tracing the uncountable nuances of musical activity.

Fig. 2 Measures 226–242 of Ponce's *Sonata no. 2*, first movement.

Fig. 3 E Aeolian to $G_\#$ Aeolian, passing through E Lydian.

References: [1] Bates, I. & R.C. Veltkamp "A Geometrical Distance Measure for Determining the Similarity of Musical Harmony". Technical Report UU-CS-2011-015, May 2011, Department of Information and Computing Sciences, Utrecht University, Utrecht, The Netherlands. [2] Bates, I. "Vaughan Williams's five variants of "Dives and Lazarus": a study of the composer's approach to diatonic organization". *Music Theory Spectrum*, 34 (1) : 34–50. 2012. [3] Guerra, D. "Manuel M. Ponce: a study of his solo piano works and his relationship to Mexican musical nationalism". Dissertation. UMI Microform 9722750 Copyright 1997, by UMI Company, Ann Arbor, Michigan. [4] Hook, J. "Contemporary methods in mathematical music theory: a comparative case study". *Journal of Mathematics and Music*, 7 (2) : 89–102, July 2013. [5] Hook, J. "Signature transformations". In: Douthett, J., M. Hyde, C. Smith (Eds.) *Music Theory and Mathematics: Chords, Collections, and Transformations*. Rochester, NY: University of Rochester Press, 2008. pp. 137–160. [6] Pearsell, E. *Twentieth Century Music Theory and Practice*. Routledge, New York, 2012. [7] Ponce, M.M. *Sonata No. 2 for piano*, New York, Peer International Corporation, 1968.

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"Compositional Generation of Macro-Structures with Dynamical Systems in my Opera *Dunaxhii*"

Keywords: improvisation, CGMD, movement, gesture, dynamics in composition.

Abstract: Skilled improvisers are able to shape in real time a music discourse by continuously modulating pitch, rhythm, tempo and loudness to communicate high level information such as musical structures and emotion. Interaction between musicians, corresponds to their cultural background, subjective reaction around the generated material and their capabilities to resolve in their own terms the aesthetics of the resultant pieces. In this paper I introduce CGMD, a multi-platform environment, which incorporates music and movement gestures from an improviser to acquire precise data and react in a similar way as an improviser.

In this case CGMD takes samples from the Zapotec language's rhythms, as well as from a particular improviser, and learns for each style a probabilistic transition automaton that considers gestures to predict the most probable next state of the Zapotec rhythm and musician style. In this paper I will demonstrate CGMD's flexibility and potential to generate macro-structures in my opera *Dunaxhii* in three acts for soprano, contra-tenor, baritone, chamber ensemble and live electronics.

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"Textural Contour: A Proposal for Textural Hierarchy through the Ranking of Partitions Lexset"

Keywords: Musical Contour Theory, Theory of Integer Partitions, partitional analysis, musical texture, ranking.

Abstract: This paper proposes a new application of the Musical Contour Theory (MCT), departing from the conception of Michael Friedmann (1985) and Robert Morris (1987, 1993), by extending its principles to the music textural domain. The MCT consists of a numerical abstraction of levels, ordered from 0 (lowest level) up to n-1 (where n is the number of different levels in the structure), describing the relative position among all levels according to some criterion. For example, a melodic contour noted < 0 2 1 > indicates a motive that begins at the lower pitch ascends to the highest one, ending at the intermediate one, with no concern about absolute pitches involved. This abstraction allows the establishment of relations among distinct motives based on its identity or related transformational process by application of canonical operations (inversion, retrograde, and retrograded inversion). This abstraction also enables the generation of derived contour using mathematic processes and describes information about contour's structure. In spite of historical focus of MCT on the pitch, there are several studies that deal with other structural parameters using the same principle of abstraction. The main interest of the present paper lies in the measurement and comparison of distinct textures according to Wallace Berry's (1976) definition of texture: a set of interactions and interrelations between sounding components. The methodology is related to the parameters of Partitional Analysis (henceforward *PA*; see Gentil-Nunes, 2009). PA is a new analytical tool for examining musical textures by combining textural analysis of Berry and the Theory of Integer Partitions (Andrews, 1984). This approach leads to some concepts that formalize the textural organization and their progression by numerical representations of musical concurrent ideas. Berry proposed a formal methodology for demonstrates textural configurations constructed from the comparison of basic features, like rhythmic profiles and contour, of the different sounding components. These configurations are read as integer partitions in PA. From observation of similarity and contrast of the components that constitute a textural configuration it is possible to establish a partition that summarizes the relations of musical elements. For example, three instruments may be organized following the partitions of number "3": a three-part block [3], two-part block and a solo [2+1], and three-part polyphony [1+1+1]. From the assessing of binary relations inside the configurations, Gentil-Nunes proposes a pair of indices that expresses its internal relations: *agglomeration* (*a*), which concerns the thickening of the internal elements, defined by the collaboration between its elements (sound blocks); and *dispersion* (*d*), which concerns the internal diversity, defined by the contrast between its elements (polyphony).

Gentil-Nunes also proposes the *partitional operators*, which express the process of internal transformation involved in the progression of one partition to the next. The partitional operators are classified as positive or negative, according to the progressive or recessive characteristic of the transformations, and are classified, by the present author, into three groups: (1) *simples*, (2) *compounds* and (3) *relational*. In this paper only the *simple operators* are focused: *resizing*, *revariance* and *simple transfer*. *Resizing* (*m*) is an operator derived from the inclusion relation, where the antecedent partition is contained in the consequent, and its occurrence concerns an increment or decrement of one of the elements of the partition (relative to the thickness). For example, the partition [2] results in partition [3] using *m+* and go to [1] using *m–*. *Revariance* (*v*) is also an operator derived

from the inclusion relation and its occurrence concerns the addition or subtraction of a new component of density-number 1 ("density number" is Berry's term to refer the total number of simultaneous voices or lines), changing the degree of polyphony. For example, the partition [1 2] results in a partition [1 1 2] using *v*+ and go to [2] using *v*–. *Simple transfer* (*t*) derives from the use of both *resizing* and *revariance* operations in compensatory movements, resulting in a constant number of sounding factors originate from the internal reorganization of components in both thickness of the parts and number of parts. The density-number on *simple transfer* is constant, which implies that the positive and negative notions are based on common practice, where polyphonic partitions are considered as more complex than partitions massive ones and this difference of degree of complexity determines the movement of it. For example, the partition [1 2] results in a partition [1 1 1] using *t*+ and in [3] using *t*–. From the adjacent of the partitional operators' relations, a hierarchy for each individual process is established, resulting in a textural configuration taxonomy, using the Hasse Diagram based on the *Partitional Young Lattice* (PYL). The PYL is an abstraction that encompasses all partitions (with their correspondent indices *a*, *d*) from 1 to a given number, organized by partitional operators involved on the partitions lexset (collection of lexical set formed by all partitions from 1 to the number of sounding factors involved). This organization allows the achievement of a *textural contour* by applying the abstraction of MCT on the PA's concepts, considering the relative textural complexity of the partitions from the simplest to the most complex. Through this abstraction, one can compare two apparently different textural progressions, relating them to the same textural contour that can be used both as an analytical tool and as a compositional resource. The partitions form a partially ordered set, i.e., the ranking is not linear.

This paper proposes a ranking based on binary comparisons between partitions, using the common practice found at the textural vocabulary (like the *simple transfer* orientation) and the partitional operators as a guide. Each operator drawn a path of partitions with its own ranking based on the orientation of movement. The methodological proposal by Ryszard Janicki (2008) for ranking comparison inside partial orders is also used as reference. The relative complexity level of each partition in a textural progression is defined by the partitional operators involved. If the progression occurs in a single partitional operator line, the hierarchy forms a linear ranking, but most of musical textural progressions uses a group of partitions that belongs to more than one partitional operator path and, therefore, the linear ranking is not possible in these cases. Some partitions pairs are incomparable. For example, the partitions [2] and [1 1] are founded at *simple transfer* path and the analysis based on *simple transfer* concept reveals that [1 1] is more complex than [2]. The partitions [2] and [3] belongs to the same *resizing* path. The analysis reveals that [3] is also more complex than [2]. The partitions [1 1] and [3] do not have any particular path in common, which implies that they are incomparable. The ranking proposed in this study examines these relations and proposes a linear order of levels to the weak order partitions on the PYL based on musical common practice. The level of complexity of incomparable partitions is numbered inside the same order level but with a decimal number that indicates the subtler difference of the amount of real components (group of congruent elements on partition). For example, on the partition progression [1 1] [2] [3], the textural contour will be < 1.2 0 1.1 >. The proposal for decimal numbers instead of using only integers as in MTC abstraction is intended to reveal that: (1) the textural progression has incomparable partitions; (2) the density-number relation between the incomparable partitions and (3) the structural relation between the real components of partitions that share the same level of complexity, as example below. The paper concludes with an example of methodological application of the textural contour and ranking partitions.

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PLENARY LECTURE:

"The Sense of *Subdominant*: A Fregean Perspective on Music-theoretical Conceptualization"

Keywords: diatonicity, formal concept analysis, well-formed scales, diazeuctic scales, structural modes, sense and reference in music.

Abstract: This case study in mathematical conceptualization investigates selected aspects of the music-theoretical meaning of the term *subdominant*. The subject will be approached from three sides. Gottlob Frege's distinction between *sense* and *reference* as two types of meaning serves as a unifying idea for this investigation. (In a celebrated example Frege points out, that the expressions "morning star" and "evening star" have the same referent; namely the planet Venus. And he observes that the proposition "morning star" = "morning star" is trivially true, while the proposition "morning star" = "evening star" expresses a true insight. Therefore, he argues, the sense of "morning star" must be different from the sense of "evening star").

(1) The term *subdominant* lends itself in a special way for an illustration of that distinction. Based on the historical meanings of the term *subdominant* a new concept is introduced and investigated: *the diazeuctic scale,* a well-formedly generated scale modulo octave, whose diazeuxis (the difference interval between generator and cogenerator) is a step interval. This concept encompasses precisely those scales where the two *senses* of *subdominant* intersect: Jean-François Dandrieu (1719) suggests a mode of presentation where reference is being made to the scale degree below the dominant. Jean-Phillippe Rameau (1726) adopts the term in order to underlay it with a different mode of presentation, namely as a scale degree a fifth below the tonic. Both senses are instances of complementary paradigmatic tone relations and deserve to be studied in combination. In analysis they interact with syntagmatic relations such as *pre-dominant* or *post-tonic* (e.g. in post-cadential overshooting).

(2) Mathematically equivalent (or closely related) concepts are revisited: Eytan Agmon (1989): *diatonic scale*, John Clough and Jack Douthett (1991): *hyperdiatonic scale*. I argue that, and illustrate how, the detection of such equivalences augments the amount of music-theoretical knowledge. *Formal Concept Analysis* (Ganter& Wille 1999) serves as a theoretical and methodological background for revisiting the scale taxonomy by Clough et al. (1999). I argue, that the detailed distinction of different senses (modes of presentation) on the paradigmatic level, provides a useful refinement of the conceptual inventory for analytical purposes.

(3) I discuss the meaning of *subdominant* in the context of the *structural modes* as proposed by Karst de Jong and Thomas Noll (2011). The underlying structural scale is the smallest instance of a diazeuctic scale. De Jong and Noll call the three scale degrees in each of these three modes *tonic*, *subdominant* and *dominant* and thereby interpret the three tonal functions in purely scale-theoretic terms. Their emphasis of the dual articulation of tonerelations through structural steps (M2 and P4) and structural shifts (P5 and P4) is reminiscent of the two senses of *subdominant* as described in (1). The *subdominant* in the first structural mode P4-M2-P4 also behaves as in (1). But the *subdominant* in the second structural mode M2-P4-P4 (typical for II V I progressions) has a fifth contiguity with the dominant rather than the tonic. These modal refinements will be carried out.

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"How Learned Patterns Allow Artist-Level Improvisers to Focus on Planning and Interaction During Improvisation"

Keywords: improvisation, jazz, pattern, rules, corpus research, computer modeling.

Abstract: Improvisation is a component of musical practice across idioms and cultures, however, the cognitive mechanisms underlying extemporaneous musical performance are not well understood. Specifically, the function of learned auditory and motor patterns is hotly debated [see: 1, 3, 6). Here I summarize the findings from a series of studies in which we sought evidence of the cognitive basis for improvisation in solo performance settings using qualitative [2], quantitative [4], computer modeling [5], and electroencephalography (EEG) paradigms. In interviews conducted with expert jazz improvisers about solos they had just completed, participants characterized a process in which their thoughts were primarily engaged in planning and evaluating larger architectural structures with moment-to-moment decisions seemingly determined through learned procedures enacted outside of conscious awareness [2]. To further investigate the contribution of non-conscious processes to the moment-to-moment decisions of skilled musical improvisers, we examined the ability of advanced jazz pianists to improvise while completing a secondary, nonmusical counting task. We found an increased use of learned auditory and motor patterns when participants were engaged in the secondary task [4]. In a separate computer modeling project, we created an algorithm capable of creating a novel musical output based on patterns extracted from a given corpus [5]. Finally, using an EEG paradigm we recorded evoked potentials during a short improvisatory task (results of this study will be available for this presentation). Taken together, our findings demonstrate that expertise in jazz improvisation is supported by an implicit process for generating music that effectively lowers demands for consciously-mediated control. Furthermore, it appears that the use of learned patterns is central to this mechanism. We believe this automatic generative process frees the advanced improviser to concurrently focus on larger architectural structures and interaction with other ensemble members.

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Paredes-Bárcenas, Goretti & Jesús A. Torres
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"Comparison of Empirical and Specific Methods to Evaluate if the Construction of Free Plates of a Violin are Already Finished"

Keywords: luthier, violin, resonance frequencies, vibration modes, measurements.

Abstract: Traditionally, violin makers have paid great attention to the vibrations of free top and back plates of their instruments. However, due to variations in the wood, even between two adjacent pieces of the same tree, cannot copy each measurement in the parts of an instrument with good sound and create another instrument with the same qualities of the original one. In the way to recreate a good violin, what does matter is not simply the geometric measurement, but also one must involve measurements related to the vibrational properties of wood.
Often—at least in the Mexican context—the vibrational behavior of plates is not deeply enough studied by luthiers; however, such study is necessary to gain a more accurate control over planned results. As a standard guide applied during the construction of musical instruments, this analytical and constructional process can be used to estimate modifications to achieve desired results. Therefore, a more precise measurement protocol for the construction of plate-instruments would contribute to systematize and improve these constructional processes.
Many specialists have participated investigating vibrations on violin plates, since 19th century when Savart started reporting some related experiments. During 20th century, Hutchins (1983) led a group of scientist in order to study free top plates

before assembly in several instruments. His findings have been corroborated by more recent research. Moral and Jansson [see: 2] reported that *free modes* are different to *modes of assembled instruments*, but similarities are clear between modes 2 and 5 of the free tops with C3 and C4 mode of assembled instruments.
This paper shows that when free plates are evaluated by specific methods measuring vibrations of the plate, and by empirically "feeling" the plate, the results are equivalent. In other words, systematic measurement matches with the empirical evaluation of luthiers. However measurements can be taught in a simple and systematic way, so that everyone—even without knowledge on the luthiers' subjects—can apply it, while the empirical method involves a subjective evaluation that depends on the "feeling" of a teacher, as well as on other specialized experience on the subject.

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Welcome lecture (1):

"A Survey on the Mexican Tradition of Music and Mathematics"

Keywords: Mexico's history, music, mathematics.

Abstract: This is a minimal introduction on the history of the Mexican tradition of music and mathematics. The goal of this short lecture is to share with the audience a cultural basic framework for understanding the significance of this Congress in a national and international context. Although the subject is extremely rich, it will be limited to a selection of historic highlights including aspects of ancient and modern Mexico. This survey stops at the last quarter of the 20th century. The continuation of this history will be elaborated during the development of this Congress, by several lectures, mainly those registered within our special panel on Julián Carrillo.

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Special panel "Mathematics and Aesthetics in Julian Carrillo's (1875-1965) work":

"Carrillo's *vs.* Novaro's Tuning Systems Nested within the Arnold Tongues"

Keywords: Carrillo, Novaro, tuning, scaling, microtonality, Farey trees, Arnold tongues.

Abstract: Farey trees and Arnold tongues have been suggested [1:354–371] as dynamical means for organizing self-similarity scaling in music. For analytic and comparative-descriptive purposes, this lecture adapts Arnold tongues for the study of the divergences between Carrillo's and Novaro's tuning systems nested within the Arnold tongues.

1. *Introduction*

It was until late 20th century, when both philosophy of science and applied mathematics within the new dynamic paradigm, agreed to pay attention to perceptual systems as systems of knowledge generation; and vice versa, the *evolutionary preceptual* systems as systems of perception. This idea has its foundations in recent findings on information theory, genetics, linguistics and the study of emerging patterns [e.g. see, 1, 2, 3, 4], and is probably due to a paradigmatic connection of thermodynamics with new branches of science patronized by physics and mathematics. Music and musicology are beneficiaries of this, since both may easily be conceived as the practice and study of a certain kind of dynamical systems. Under this conception, the "sonic atom", timbral roughness, and generation and transformation of *musical scales*—in its broadest sense—, *forms* and *structures*, can be studied as aspects of a same phenomenon.

2. *Arnold tongues, diatonicism and multi-scalar dynamics*

This approach to musical complexity may serve to explain how endorhythms contribute to perform and elaborate musical structuration and meaning. Human and in general living organisms' endorhythms seem to be *musical* or *pre-musical* features of an evolutionary society (an idea prevailing in many musicians, from pre-Classicism to nowadays theorist and composer Erv Wilson); thus, "life" with its *rhythmic* and *harmonic* interpretations would be rather phases of a same, unique physical process.

The 19th century concept of *Farey trees*—a simple arrangement of numerical self-structuration—is closely related to many other self-referential algebras studied by modern mathematics. Introducing a differential approach to successions within these arrangements may lead to emerging patterns with "new" features and very diverse behaviour. As a mathematical device, the Arnold tongues fulfill this approach, as it is analogical to essential self-structuring patterns in living organisms codification (e.g. basic recursive genetics) and more sophisticated self-structuring grammars in verbal and non-verbal communication. Even the (Peirce)–Schenker–Lerdahl cognitive constraints of musical systems and hierarchical self-structuring, reasonably fit within this *tree-model* which is not necessarily triadic or *n*-adic, but *n*-layered, *n*-hierarchical (or *rhizomatic*), *dense* and *self-contained*. The embedding of recursive grammars within themselves, e.g. chromaticism whithin diatonicism, and Diatonicism (let this concept be useful for an algebraic-geometric definition, and not especially for the naif one) can be fruitfully associated to a multi-scalar dynamics of music.

3. *Carrillo vs. Novaro in a dynamical context*

Since the identification of *initial conditions* of a system is usually a basic start point of any dynamical system, the dynamical modellation of music must somehow be of an analogic (i.e. proportional, synecdochic) nature. From the viewpoint of this proposal, the *initial conditions* of a musical system must be explicitly related to *initial rules* (i.e. possible relationships within a universe), such as well-formedness of a pitch scale, or rhythmic proportionality (motivic structure, metre, phraseology). This still valid for any function within a musical grammar regardless its order of complexity, and also is closely related to the emergence of musical Gestalt, in its turn crucial for *meaning* of a "grammar in context" (leading to pragmatics).

Julian Carrillo (1875–1965) and Augusto Novaro (1891–1960) [see: 6, 7, 8] are pioneering—at first glance opposed—theorists in the development of tunings and scalar systematization beyond *classical tonality*. Nevertheless both of them pursue a musical grammar based on their findings of physical tuning. [8] compares their tuning systems, albeit without an analogic context for analytic direct comparison, due to the profound theoretical differences between Carrillo and Novaro. Instead, within a dynamical context, these and other "microtonal" grammaticalities may be analogically described, analyzed and used for self-structuring, when nested whithin the Arnold tongues. For this goal, *irrational grammars* (i.e. not based on Pythagorean or similar ratios), such as Carrillo's one, must be syntesized as an approach to an Arnold set—an exercise indeed comparable to any musical approach to an *ideal tuning* or an *ideal grammar*. This lecture will discuss the features of these different systems as nested within the Arnold set, and the Arnold tongues as a useful tool for musicology and music composition.

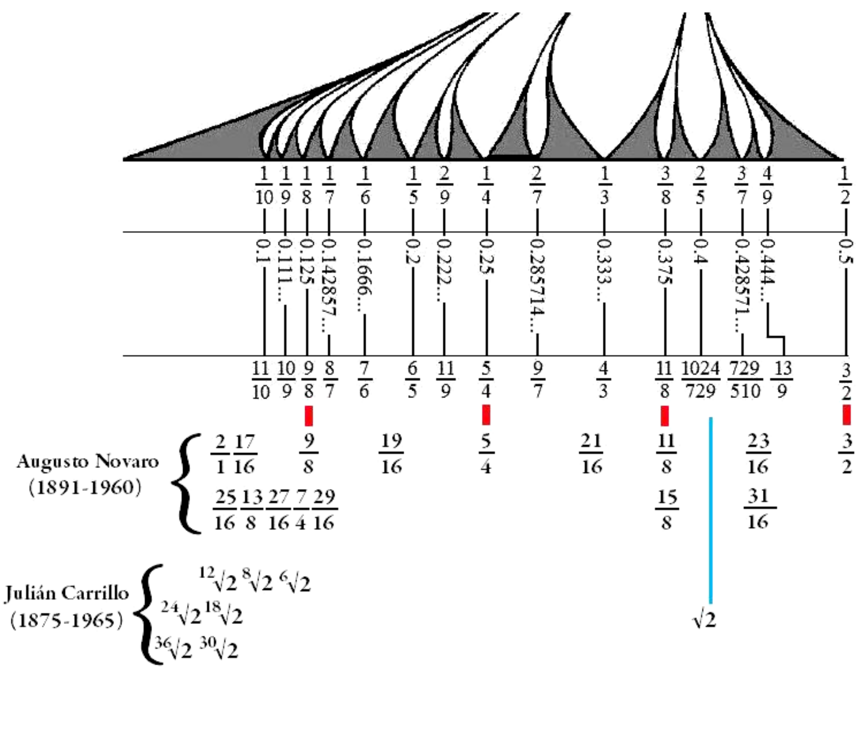

Illustration in the previous page: Arnold tongues from a differential equation first published in [5],

$$\theta_{i+1} = \theta_i + \Omega - \frac{K}{2\pi} \sin(2\pi\theta_i),$$

here adapted as a harmonicity chart mod $\frac{1}{2}$ for historical musical ratios, including (in its lowest part) Novaro's ratios and Carrillo's succesive collection of intervals obtained from $\sqrt[k_i]{2}$ series.

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"A Fuzzy Logic Approach of High Level Musical Features for Automated Composition Systems"

Keywords: Fuzzy inductive reasoning, musical coherence, algorithmic composition, musical representation, musical features.

Abstract: Automated algorithmic composition systems are now well-understood and documented [see: 1, 3]. However, their capacity for accurately manage high level musical features such as *coherence, emotion* or *personality*, is still object of discussion [6]. On the search for designing more effective systems that exhibit greater expressiveness, latest attempts have proceeded by extracting representations of those features. However, these representations appear commonly as a side effect of the research made in machine learning for the construction of composition or interactive systems. The fact that machine learning processes can effectively capture such features to a higher degree is still unclear and much research remains to be done in this area. Moreover, designed systems have not explicitly incorporated perception and semantics of the generated music, excluding the psychological sensation of the musical form perceived by the listener. Attempts to do this often deal with machine listening techniques that need high computational capacity, and the modules designed for the evaluation and adjustment of the outputs commonly operate by using a pre-established, symbolic domain [2]. A usual example is the use of fitness functions in genetic algorithms to modify the outputs until the desired ones are obtained.

In the present work, a fuzzy system approach for modeling high level features for automated algorithmic composition is discussed. Fuzzy systems are based on fuzzy logic theory [16], that deals with objects that are approximate rather than fixed and exact. Fuzzy systems require less amount of resources for processing—useful in real time situations—making them more portable, and are not restricted to pre-established structures for the evaluation modules, allowing systems to include human (and its associated psychological perspective within the design or the evaluation cycle) without having a predefined representation process of the desired output. This allows the system to extract the musical representation of the expert experience and translate it in terms of combinations of variables and features, and then, for example, to produce consistency between musical parts, through the subjective evaluation to listen combinations.

The methodology developed incorporates *Fuzzy Inference* and *Inductive Reasoning* systems [12] to evaluate coherence between algorithmically produced musical outputs. Firstly, the generating algorithms are discussed in the context of musical style. Then, the produced outputs evaluated by experts are presented. The output and its evaluation were used as input data to train the system, and the variables that produce "coherent" outputs and the relationships among them where identified as rules. Finally, the extracted rules and results are discussed in the context of the musical form and taking into account the psychological perception. The generated rules were incorporated to modify the original algorithms and new patterns were generated and compared with the original ones by using music information retrieval tools.

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"Phase Synchronization Analysis as Fingerprint Classifier for the Teponaztli's Timbral Features"

Keywords: phase synchronization, timbre classification, teponaztli, percussion classification, idiophone.

Abstract: It has long been noticed that a wide variety of coupled oscillatory phenomena synchronize, for example the lighting of fireflies, the singing of frogs and pendulums of clocks [see: 1, 2]. Synchronization can occur regarding different features of the oscillatory phenomena, such as phase or frequency. In this work we refer to phase synchronization. The phase of an oscillator at any given time is a quantity that increases 2π in each cycle and corresponds to the fraction of a cycle which has elapsed, relative to an arbitrary point.

A particular kind of phase synchronization, when one of the oscillators leads the oscillation of the overall system due to the tendency of the system to oscillate at certain frequencies (the physical properties of the oscillators involved) is known as *resonance* [1, 3]. Specifically, our work focuses on damped resonance but the framework applies to the more general case of synchronization.

Our contribution is the analysis of the timbral features of the *teponaztli* in the context of its phase synchronization. Eventually we propose the generalization of this analysis for a variety of idiophones. The teponaztli is a slit drum native to Mexico; a unique kind of idiophone made of a single piece of carved wood, with two strips—rarely three or more—which are hit by a drumstick and produces two different pitches. We study typical teponaztli of two strips, tuned to a harmonic interval which to the Western, modern ear, falls nearby a "third or fourth" (with relative roughness, sometimes unfocused because of the quality of the wood's instrument). Given that the instrument is made of a single piece, we can conceive it as a coupled oscillatory system which may synchronize its phases at the moment of vibrating (from the attack to the end of the resonances). Our hypothesis is that the measurement of this synchronization and its evolution, reflects key features of the instrument timbre.

The experimental set up is described next. For each teponaztli, three sets of three pulses were analyzed, each set of pulses using the exact same set up and protocol except for the drumstick, which changed from *soft*, *medium*, and *hard*. For the first pulse the larger tongue was fitted with a cardioid microphone while the other one was fitted with a transducer microphone, then, the larger tongue was hit once. For the second pulse, the microphones were exchanged and this time the smaller tongue was hit. Finally, for the third pulse, both tongues featured transducers and both were hit once simultaneously. This was repeated for each kind of drumstick.

Each pulse was treated separately, but in all three cases, both, the recording of the sound produced by the vibration of the hit tongue, and the recording of the vibration of the tongue obtained via a transducer, are time series from which it is possible to obtain respective phases. The recordings were cut starting at the attack and ending, when oscillations had completely stopped. To extract the phase a complex signal is constructed via the Hilbert transform as follows; for each time series $s_j(t)$ with $j=\{1,2\}$, which correspond to each of the strips of the teponaztli, we generate a complex signal (1):

$$\zeta_j(t) = s_j(t) + i\, s_{jH}(t) \quad , \tag{1}$$

where $s_{jH}(t)$ is the Hilbert transform as in (2), that is:

$$s_{jH}(t) = \pi^{-1} \text{ P.V.} \int_{-\infty}^{\infty} \frac{s(\tau)}{t-\tau}\, d\tau \ , \tag{2}$$

where P. V. refers to the *principal value*, and from which equation (3) is obtained,

$$\zeta_j(t) = A_j(t) e^{i\varphi j(t)} . \tag{3}$$

Equation (3) yields the functions for instantaneous amplitude and phase, $A_j(t)$ and $\varphi_j(t)$, respectively.

Once the phases are obtained the synchronization index is computed. To do so, we use the stroboscopic approach and conditional probability. In this context, the synchronization index is a number between 0 and 1, where 1 is *complete synchronization* and 0 represents *total lack of synchronization*.

Each recording is divided in ten windows of the same length, and a synchronization index is calculated for each of them. Then an overall index is obtained by averaging these partial indices.

In order to calculate each index, both phase functions are mapped around a circle by taking them modulus 2π. A parameter a is selected and the interval $[0,2\pi]$ is divided in a subintervals, I_i with $i = \{1, \dots a\}$, of size $2\pi/a$ which cover the circle; a partial synchronization index is obtained for each of these subintervals. More specifically, the partial synchronization index, λ_i, represents the probability of having the phase of one of the oscillators in a certain subinterval I_i, given that the phase of the other oscillator is at that same interval, this is:

$$\lambda_i = P(\varphi_2(t) \in I_i \,|\, \varphi 1(t) \in I_i), \tag{4}$$

with t inside the time window in question.

Once the phase functions are mapped around a circle, the values of the phase of one of the oscillators (the instrument strips) are counted and recorded every time the value of the phase of the other oscillator falls inside a given interval I_i.

Let M_l be the number of occurrences of the phase of the leading oscillator that fall inside the I_l interval, and let v_l with $l = \{1, \dots, M_l\}$ be a vector containing the corresponding values of the phase of the other oscillator at those specific times; then the partial synchronization index for the *l-th* interval is

$$\lambda_1 = \left| \frac{1}{M_l} \sum_{j=1}^{M_l} e^{i\ v_i} \right| \tag{5}$$

Finally, all partial indices are averaged to get the synchronization index Λ in each time window

$$\Lambda = \frac{1}{a} \sum_{i=1}^{a} \lambda_i . \tag{6}$$

The classification of this synchronization and its evolution in time may turn into a new method for classifying timbre in idiophones which do not have conventional, fix tuning. Furthermore, this classification may contribute to analyze the relationship between timbre and self-structuring in music composed for idiophones.

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"The Musical Experience between Measurement and Computation: from Symbolic Description to Morphodynamical Approach"

Keywords: computation, measurement, musical space, set theory and algebra, spectromorphology, morphodynamics.

Abstract:

1. *Introduction*

Music and mathematics have a lot of common grounds. They both involve processes of thought, but where mathematics is concerned basically with symbols without any physical connection to the world, music has sound as its major category. Music, in this view, is characterized by a sonorous articulation over time, which can be described in physical terms. Yet, it is possible to conceive of music also at a virtual level of imagery and to carry out symbolic computations on mental replicas of the sounds. The major aim of this contribution, therefore, is to explore some basic insights from algebra, geometry and topology, which might be helpful for an operational description of the sounding music. Starting from a conception of music as a formal system, it argues for a broadening and redefinition of the concept of computation, in order to go beyond a mere syntactic conception of musical sense-making and a mere symbol-processing point of view.

2. *Experience and computation: internal and external semantics*

Computations, which take as a starting point a set of elements to operate upon, are considered mostly from a symbol-processing point of view. The basic idea is formal symbol manipulation by axiomatic rules, whith a complete conceptual separation from their physical embodiment. As such, it is by definition implementation-independent and finds its application in computer programs, which handle discrete symbols and discrete steps in rewriting them to and from memory to a sequence of rules. The steps can only be defined by a *measurement process*, and the symbols are records of a measurement. The time of measurement, further, does not proceed simultaneously with the time of the dynamics, which means that the sequence of computational steps is "rate-independent". Formal systems, in other words, must be free of all influence other than their internal syntax. To have meaning, however, they must be informally *interpreted*, *measured*, *grounded* or *selected* from the outside, which involves a transition from rate-independent programmed computation to a rate-dependent dynamic analog with measurementss proceeding in real-time [see: 8].

Dealing with music, however, is not merely symbolic modeling and computation. It also involves processes of sense-making that match the auditory input against a knowledge base and coordinate it with behavioral responses. The music user, in this view, can be considered as a learning device, made up of sensors, coordinative computations and effectors, which are related to the primitive functionalities of *measurement*, *computations* and *control* [3]. Each of these functions can be an arena for adaptation, but contemporary conceptions of learning devices have focused almost exclusively on coordinative, "cognitive" adaptation, which mostly neglects the possibility for adaptation at the level of perception and action. As such, they are in line with the current trend of syntactization of semantics, which began in the 1930s with the "logical semantics" of Carnap and the "model-theoretic semantics" of Tarski. This syntactization is accomplished by completely encoding the world, so that symbols are seen in relation to a completely logical-symbolic structure, postulating merely sets of "possible worlds" and "world-states" without having to specify any sets of observables or having to verify any truth values with respect to the external world. If the symbols are without relations to the external world, we can conceive of them in terms of "internal" semantics; if they establish a relation to the outer world, they involve an "external" or "real semantics" [2].

3. *Measurement and symbolic play*

The notion of measuring device was introduced by Hertz who pointe out the possibility of linking particular symbol states to partiucular external states of affair. A measurement, in his view is a pointer-reading of an observable that functions as the initial condition of a formal model for predicting the value of a second one. It reflects the particular interactions of the measuring apparatus with the external world and allows the device to carry out predictive arithmetic and/or logical calculations on the pointer-signs [3]. The role of symbolic play must be considered here as formal computation is carried out on the symbolic counterparts of the observables, and not on the observables themselves.

Computations are thus considered mainly from a symbol-processing point of view. There is, however, a broader conception of computation, which considers the input/output couplings, and which can be handled in terms of *modeling* or *predictive computations* [1]. Computation, in this view, embraces the whole field of mental operations, which can be performed on symbolic representations of the sounds. Applied to music, this should mean that we should conceive of musical "objects" and "processes" in terms of formal and syntactic operations, somewhat analogous to the mathematical activities as counting,

measuring, classifying, comparing, matching, ordering, grouping, patterning, sorting, labeling, inferring, modeling and symbolic representation. Cognitive operations, in this view, are internalized actions, which can be reduced to the processes of classifying, seriating, putting in correspondence and combining, which, in turn, are related to the logico-mathematical operations, which are abstractions of concrete operations as collecting, ordering and putting things together [10].

4. *Music as an algebraic structure: the concept of musical space*

The computational approach to music stresses the possibility to carry out symbolic computations on mental replicas of the sounds. It takes as a starting point a set of elements upon which to operate, which can be labeled symbolically as discrete things with unit character, but which can have a continuous representation as well. The elements, therefore, can be characterized as variable functions of time, which can be of any length, ranging from discrete focal points to larger temporal events. Their delimitation, then, can be described in algebraic terms by defining them as *variables*, taking as a starting point a domain that represents the totality of sounds. It is possible, further, to reduce this sonic world or sonic unviverse to its arithmetical substrate, and to conceive of it also as a *musical space*, consisting of a set of points, each of them corresponding to a number. Musical figures can be delimited in this space and can be considered as configurations of points, which can move from one configuration to another. The geometric space that figures as a framework for these transformations has to take account of this and must be chosen according to some criteria (every possible point must have an allocation in the space, and every transition from one configuration to another must be possible). This calls forth a dynamic conception of geometric space as a collection of points. Spaces, then, are networks within which points can be fixed by giving them some numbers, called coordinates.

Musical space, accordingly, can be conceived as a collection of elements, that can be described in a formal way as an *algebraic structure*, i.e. a non-empty set together with a collection of (at least one) operations and relations on this set. The central problem, however, is the definition of the elements, as musical space and time have to be integrated in the definition, together with set theoretical, geometrical and algebraic points of view.

5. *Musical space as topological space*

Musical space can be defined as a collection of points that constitute the domain (the arguments) upon which predication processes can be applied. The result of these processes are propositions that assign some general term to individuals. Predication, however, does not apply to points, that have no extension, but to units that are recognizable as such. At a formal level these units are systems of isolated points in one or more dimensions, somewhat comparable to the point-events of physics. It is necessary, therefore, to construct a mathematical model for the description of the physical domain (the sonorous universe) from which the units can be recruited, and for giving a numerical description of them. Applying this way of thinking to the domain of music provides an operational description of musical space as a *topological space*, which makes it possible to conceive of every musical structure as a set of elements, which can be defined by selecting sets of points. Musical configurations, then, can be defined as point-sets that can be transformed into other configurations, and this in a gradual or rather abrupt way.

In applying transformations to sets of points, the configurations mostly are left invariant with respect to at least some properties. These are called *topological invariants*. The sets, however, must be structured, allowing the mapping of each element of set A onto set B, with elements of A being the domain and elements of B being the co-domain (the values or images of A). Most interesting, however, are operation preserving mappings, that preserve the structure of the algebraic system. Such mappings or *homomorphisms* generate a transformed image of the original structure (the domain) in the image set (the range) and provide a numerical basis for identification and transformation algorithms.

6. *From static description to morphodynamical approach*

Music can be considered as a collection of sound/time phenomena, which have the potential of being structured. As such, it entails processes of discretization of the sonorous unfolding, which involve a quantal aspect of perception [7]. It allows us to describe music in geometric terms as a kind of distributed substrate with discontinuities and focal allocations of semantic weight. As such, we can conceive of music in "morphological" terms, which genuinely combines a discrete with an analogous description of the sound. It is exemplified in the conception of spectral morphology [4, 5] —where sonic morphologies may resemble one another, may be transformations of one another and may oppose one another—and in related elobarations of *spectromorphology* [11] and *acousmatic morphology* [6]. Such a morphological way of thinking is challenging. It provides a description of typical patterns of temporal unfolding as well as a description of their sounding articulation. It has received also impetus from other areas of research, to mention only the *morphological* and *morphodynamical* procedures for delimiting morphological lexicons as proposed by [9, 12].

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"Generic Additive Synthesis? Hints from the Early Foundational Crisis in Mathematics for Experiments in the Ontology of Sound"

Keywords: sound ontology, foundational crisis, philosophy of mathematics, sound synthesis, experimental programming, live coding, axiomatics.

Abstract: Already since the 19th century sound research, the idea of a frequency spectrum has been constitutive for the ontology of sound. Despite many alternatives, the sine wave thus still functions as a preferred basis of analysis and synthesis. This possibility has shaped what is taken as the most immediate and self-evident attributes of sound, be it in the form of sense-data and their temporal synthesis or the aesthetic compositional possibilities of algorithmic sound synthesis [2].

Against this background, our talk will consider the early phase of the foundational crisis in mathematics (*Krise der Anschauung* [5]), where the concept of continuity began to lose its self-evidence. This shall permit us to reread the historical link between the Fourier decomposition of an arbitrary function and early set theory [3] as a possibility to open up the limiting dichotomy between time and frequency attributes. With reference to Alain Badiou's ontological understanding of the praxis of axiomatics [1], we propose to take the search for a specific sonic situation as an experimental search for its conditions or inner logic, here in the form of an unknown decompositional basis and its consequences.

This search cannot be reduced to the task of finding the right parameters of a given formal frame. Instead, the formalisation process itself becomes a necessary part of its dialectics that unfolds at the interstices between conceptual and perceptual, synthetic and analytic moments. Generalising the simple schema of additive synthesis, we contribute an algorithmic method for experimentally opening up the question of what an attribute of sound might be, in a way that hopefully is inspiring to mathematicians, composers, and philosophers alike.

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Special panel "Mathematics and Aesthetics in Julian Carrillo's (1875-1965) work":

"Julián Carrillo's Microtonal Counterpoint"

Keywords: Julián Carrillo, Mazzola's counterpoint, Counterpoint theorem, interval dicotomies, microtonalism.

Abstract: In this talk I will show the progress made about my undergraduate thesis project under the codirection of Lluis-Puebla and Agustin-Aquino. This dissertation research consists on a study around the works of Julian Carrillo looking for paralelisms between them and the progress of the mathematical-musical theory, specially the counterpoint theorem, because the specific microtonal space where the Carrillo's works lie is somehow isomorphic to $\mathbb{Z}_{16}$, which is the least bound for exactly one interval's dicotomy. The main goal is trying to read if Carrillo knew this dicotomy in his works, and in a more general way, try to develop a method to find this counterpoint dicotomies in any composition. A later goal, if having positive results, is to verify if Carrillo knew also about the prediction rules of the mazzolian counterpoint in a microtonal context.

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"*Sonus Geometria*: A Theoretical Classification Model of Electroacoustic Concepts Based on Fundamentals of Topology Dynamics"

Keywords: topology dynamics, electroacoustic music, composition, theoretical electroacoustic math analysis.

Abstract: This paper proposes a formal model of conceptual categorization for electronic and electroacoustic music composition based on Topological Dynamics. Given the dynamic system $\{X, \mathbb{R}, \pi\}$, where X is a metric space and $\pi : X \times \mathbb{R} \to \mathbb{R}$ is a continuous function, we define by analogy, the Sound Space as a set of discrete events and whose behaviour is precisely defined by such topological system.

Applying basic definitions from usual topology and from topological dynamics such like invariant sets, limit sets, periodicity and Poisson's stability, we present a mathematical model that formalize processes and compositional concepts. Pieces which might include: acoustic instruments with electronics, multichannel spatialization, timbral and spectral dynamics and any electroacoustic compositional processes, can be structured in formal mathematical terms. Thereby gathering a formal and stable bridge between language and concept in electroacoustic compositions and the field of topological dynamics. This model allows a well founded theoretical analysis of electroacoustic works such as the one we will present with the piece *Murmullo a voces*.

Sonus Geometria works as an alternative for well fundamented analysis tool in electroacoustic compositions due to its topological approach and oriented consistency balance between the theoretical baggage and final work's aesthetics. Furthermore, the model allows us to use it as a sources for algorithmic processes in electroacoustic composition, particularly those funded in theoretical dynamic processes in the field of Dynamic Topology.

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"Melodic Pattern Segmentation of Polyphonic Music as a Set Partitioning Problem"

Keywords: polyphonic music, motif, equivalence class, segmentation, integer programming.

Abstract: In polyphonic music, melodic patterns (motifs) are frequently imitated or repeated, and transformed versions of motifs such as inversion, retrograde, augmentations, diminutions often appear. Assuming that economical efficiency of reusing motifs is a fundamental principle of polyphonic music, we propose a new method of analyzing a polyphonic piece that economically divides it into repeated motifs.

To realize this, we take an integer programming-based approach and formalize this problem as a set partitioning problem, a well-known optimization problem. This analysis will be helpful for clarifying the structures of polyphonic music and may be useful for some systems of music analysis, performance, and composition.

1. *Motif division*

In polyphonic music like fugue-style pieces or J.S. Bach's *Inventions* and *Sinfonias*, melodic patterns (*motifs*) are frequently imitated or repeated. Although some motifs are easy to find, others are not. This is because they often appear implicitly and/or appear in the transformed versions such as inversion, retrograde, augmentations, diminutions. Therefore, motif analysis is useful to understand how polyphonic music is composed.

Simply speaking, we can consider the motifs that appear in a musical piece to be *economical* if the number of species of motifs is small, the number of repetitions is large, and the lengths of the motifs are long. Assuming that this economical efficiency of motifs is a fundamental principle of polyphonic music, we propose a new method of analyzing a polyphonic piece that efficiently divides it into a small number of species of motifs. Using this division, the whole piece is reconstructed with the pieces of motifs like a jigsaw puzzle (we call such a segmentation a *motif division*). This analysis will be helpful to clarify the structures of polyphonic music and may be useful for some systems of music analysis, performance, and composition.

Studies about finding boundaries of melodic phrases are often based on human cognition. For example, [2] is based on grouping principles of Gestalt psychology, and [3] is based on short memory model. While these studies deal with relatively short range of perception and require small amounts of computational time, we focus on global configuration of motifs on the level of compositional plan. This requires us to solve a global optimization problem that is hard to solve. To deal with this difficulty, we take an integer programming-based approach [5] and formalize this problem as a set partitioning problem [1]. This problem can be solved by IP-solvers that use efficient algorithms such as the *branch and bound* method.

2. *Transformation Group and Equivalence Classes of Motifs*

In this section, we introduce equivalence classes of motifs derived from a group of motif transformations as the criterion for the identicalness of motifs. These equivalence classes are used to formulate the motif division in Sect. 3.

Firstly, a motif is defined as an ordered collection of notes $[N_1, N_2, \ldots, N_k]$ ($k > 0$), where N_i is the information for the i^{th} note, comprising the combination of the pitch p_i, start position s_i, and end position e_i ($N_i = (p_i, s_i, e_i)$, $s_i < e_i \leq s_{i+1}$). Next, let $\mathscr{M}$ be the set of every possible motif, and let T_p, S_t, R, I, A_r be one-to-one *mappings* (transformations) from $\mathscr{M}$ to $\mathscr{M}$, where T_p is the transposition by pitch interval p, S_t is the shift by time interval t ($p, t \in \mathbb{R}$), R is the retrograde, I is the inversion, and $A_r (r > 0)$ is the r-fold argumentation (diminution, in the case of $0 < r < 1$). These transformations generate a transformation group $\mathscr{T}$ whose operation is the composition of two transformations and whose identity element is the transformation that does noting. Each transformation in $\mathscr{T}$ is a *strict imitation* that preserves the internal structures of the motifs.

Here, a binary relation between a motif m ($\in \mathscr{M}$) and $\tau(m)$ ($\tau \in \mathscr{T}$) can be defined. Due to the group structure of $\mathscr{T}$, this relation is an equivalence relation (i.e., it satisfies reflexivity, symmetry and transitivity). Then, it derives equivalence classes in $\mathscr{M}$. Because the motifs that belong to the same equivalence class share the same motif structure, thay can be regarded as identical (or from the same species)[1].

3. *Formulation as a Set Partitioning Problem*

A Set partitioning problem, which has many applications in the context of operations research, is an optimization problem defined as follows. Let N be a set that consists of n elements $\{N_1, N_2, \ldots, N_n\}$, and let M be a family of sets $\{M_1, M_2, \ldots, M_m\}$, where each M_j is a subset of N. If $\cup_{j \in X} M_j = N$ is satisfied, X, a subset of indices of M, is called a *cover*, and the cover X is called a *partition* if $M_{j1} \cap M_{j2} = \emptyset$ is satisfied for different $j_1, j_2 \in X$. If a constant c_j called a *cost* is defined for each M_j, the problem of finding the partition X that minimizes the sum of the costs $\Sigma_{j \in X} c_j$ is called *a set partitioning* problem.

If N_i corresponds to each note of a musical piece to be analyzed and M_j corresponds to a motif, the problem to find the most efficient motif division can be interpreted as a set partitioning problem. The index i starts from the first note of a voice to the last note of the voice, and from the first voice to the last voice. M_j ($1 \le j \le m$) corresponds to $[N_1]$, $[N_1, N_2]$, $[N_1, N_2, N_3]$, …, $[N_2]$, $[N_2, N_3]$, … in this order. The number of notes in a motif is less than a certain limit number. The *objective function* is defined as the number of motifs that appear in the motif division (this is realized if $c_j = 1$). This objective function is inversely proportional to the average number of notes (*length*) for the motifs that appear in the motif division. Also, additional constraints are introduced to control the motif division adequately. For example, the number of equivalence classes that appear in the motif division is fixed to a certain small number (thanks to this constraint, repetition of the motifs is facilitated). By this formulation, we can expect an economical solution mentioned in Sect. 1. Fig.1 shows an example of the set of motif classes in Bach's *Invention No. 1*, obtained by solving this problem using the *IP-solver Numerical Optimizer* 16.1.0.

Fig. 1 An example of the set of representatives of motif classes that apper in the motif division of J.S. Bach's *Invention No. 1* (the number of motif classes was set at 13).

Notes:

[1] Although the criterion for identical motifs defined here only deals with strict imitations, we can define the criterion in different ways to allow more flexible imitations, such as by (1) defining an equivalence relation from the equality of a shape type [see: 4] and (2) defining a similarity measure and performing a motif clustering (the resulting clusters derive an equivalence relation). In any case, making equivalence classes from a certain equivalence relation is a versatile way to define the identicalness of the motifs.

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"Diagrams, Games and Time"

Keywords: time, indeterminacy, games, graphs, diagrams, actions, Tom Johnson's *Networks and Looking At Numbers*, Christian Wolff's *For One, Two or Three People.*

Abstract: In the closing section of his essay *Towards a Philosophy of Music called Destiny's Indicators*, Iannis Xenakis proposes an extraordinarily audacious program for music: to alter for the human mind the transcendent categories of time and space as such. "Consequences: 1. It would be necessary to change the ordered structures of time and space, those of logic, ... 2. Art, and sciences annexed to it, should realize this mutation." However, in theorizing time Xenakis retained many standard notions, particularly that of a linear ordering of events. Yet to think of time as a linear sequence from past to future does not present the only way of conceiving temporality (one may for example think of Bergson's conic model, where the important dynamic rather goes between coexisting levels of the past, contracting towards and distending away from the present).

In Xenakis's thinking, the non-linear is relegated entirely to the complementary category of 'music outside-time'. This is associated with various algebraic and other structures, such as symmetry groups which are made to operate on parametrized musical objects, allowing for definitions of transformations and relations between gestures in-time. Such groups can be thought of as generated by a base set of permutations, and this can make for a representation of the operations in graph form, with each vertex corresponding to a state of the musical object, and each edge to a basic operation. This comes very close to the networks that have been explored by Tom Johnson in his recent work, and in his collaboration with Franck

Jedrzejewski, *Looking at Numbers*. The typical compositional problem that Johnson addresses in his network pieces involves finding a "logical" path through these networks. The task is not obvious: the networks in themselves do not generally show a logical preference for one path over another. Put differently, these networks are not trivially linearizable, which would correspond to Xenakis's notion of such structures forming a "music outside-time".

I propose to see such representations as rather encoding a richer, non-linear approach to time. A graph (or other suitable mathematical category) represents a whole field of potential time paths, which can be projected into linear time in many ways. It is my poetical hypothesis that the form of this field is a real part of musical performance. As I put it in my 2013 essay, *Action Time*: "Time becomes a field of variations rather than a line. Of course, in practical performance, there will always be a single line drawn through this field, since the time that we live through remains linear. Yet the other variations can somehow be sensed. They are, as paths not taken, part of the performers' actions and determine a quality of performing. They thereby remain active as virtual parts."

This notion of a *time field* is tied to what the same essay termed an *action grammar*. Clearly, the two notions are interrelated: a time field is some sort of web of states, which are related by the way actions can link one state to another. The time field can also be understood as something like the frame of a Kripke model, with the action grammar defining its accessibility relations. A third type of element would be a kind of game rule in performance. This is an instance of what I will call *transcendent criteria* by which the actions could be bound, not only by their local possibilities (accessible actions in a particular state), but also by their collective effect (criteria for well-formedness of an actualized path).

It is not my present intention as a composer/theorist to present a definitive formalization of these notions. Rather, I would like to explore the questions that are musically relevant and that might possibly prove mathematically fertile. To this end, during this presentation I will explore the relations between the three notions of time field, action grammar, and transcendent criteria in two musical settings.

On the one hand, I will briefly explore my extremely simple prose composition *Ensemble*. By a very simple rule, this piece defines a time field of transitions between possible sonorities, that is surprisingly complex to navigate for an ensemble. The action grammar here is defined implicitly by the time field and by transcendent criteria.

For the second example, I will propose a reading of the notation in Christian Wolff's score *For One, Two or Three People* as constituting an action grammar. The score defines musical gestures very loosely, as a kind of very abstract diagrams for actions, rather than as specific materials. I will call this notational style *Wolff diagrams*. The important thing is that the gestures are defined partially, that is, in relation to other sound events that are not co-determined by the notation, and that the gestures must be coordinated with. Performers play a given repertoire of gestures in a free order, trying to comply with the rules on the spot, which gives the piece a musically complex, instable, game-like tension. That is, the challenge for musicians is to assemble the *diagrams* in real-time into a *correct* performance.

It is my hypothesis that this notational practice still has more compositional and combinatorial potential to yield. Musically, I am convinced that the Wolff diagrams have not nearly been exhausted in his original work, and that whole worlds of interactive music practices remain to be found in them. Mathematically, there may be interesting questions to be asked about the relationship between the definition of an action grammar, the universe of its correct realizations, and the properties of the time field generated, as well as their interaction with transcendent criteria. Philosophically, the hope is that such explorations will have something to say about the relationship between actions, (inter-)subjectivity, and temporality.

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"On Minimal Change Codes for Generating Music"

Keywords: minimal change codes, de Bruijn sequences, Gray codes, generative music.

Abstract: In this paper, we discuss two types of minimal change codes and their application to music composition: Gray codes, which are enumerations of all binary words of a given length *n* such that only one bit changes from word to word; and de Bruijn sequences, which are cyclic sequences that given an alphabet, represent all words of a given size exactly once. For musical purposes, we apply additional constraints. In particular, we define a maximally balanced, maximally uniform run-length Gray code and a De Bruijn sequence we denote as 'space-limited', which is constructed from an alphabet of integers that sum to 0 and where the difference of extremal values of the running sum of the sequence is as small as possible. Based on these extra constraints, we pose two open problems that might be of interest to mathematicians: 1) what are the bounds on the minimum range of run-lengths given a balanced Gray code of *n* bits, and 2) what are the lower bounds on the range of extremal values in the running sum of optimal solutions for space-limited De Bruijn sequences?

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"Restoring the Structural Status of Keys through DFT Phase Space"

Keywords: Discrete Fourier transform, Schenker, Voice leading, Geometry, Keys.

Abstract: Beethoven's *Heiliger Dankgesang*, the third movement of his late A minor String Quartet, op. 132, remains inspiring yet enigmatic nearly two centuries after its composition. Its biographic resonances, play of musical topics, and misprision of antiquated contrapuntal styles have all been well explored. But current theories of harmony are not well tooled to address one of its most puzzling features, the status of tonality in this nominally "Lydian mode" work. The Discrete Fourier Transform (DFT) on pcsets, developed by Lewin [3], Quinn [4], Amiot [1], Amiot and Sethares [2], and Yust [7] may have the potential to breach lingering gaps between current theories of tonality and the traditional notion of keys, giving hermeneutic access to the tonality of this work.

As a voice-leading based approach that can address large-scale tonal structure, Schenkerian theory is widely regarded to be amongst the most sophisticated extant theories of tonality. However, when Schenker claimed that his theory of levels would supplant traditional notions of form and key, he overplayed his hand, creating conceptual tensions that persist in Schenkerian theory today. Schachter's [5] insightful deconstruction of the Schenkerian perspective on keys stops short of denying their reality even as he claims that Schenkerian structures override them. The problem with this idea is evident in some of Schenker's own analyses. His analysis of the exposition of Brahms's F major Cello Sonata [6, Fig. 110d2], for example, incorrectly interprets the entire subordinate theme (mm. 34–60) in the key of A minor. It is actually almost entirely in the key of C major, including an elaborate C major cadential process which is diverted into an A minor cadence only at the last moment. Since Schenker's middlegrounds are reductive voice-leading progressions that retain only chords, not their tonal contexts, he recasts the tonal context of an entire thematic area implausibly in terms of what is, in this case, a rather fleeting goal.

The *Heiliger Dankgesang* presents a different, but related, kind of problem, a non-tonic conclusion indicative of a deeply entrenched disjunction between chord and key. The piece begins *in* F major, but ends only *on* F major, because its tonal context has shifted to C major. The meaning and purpose of this unusual tonal design is inaccessible to a theory that reduces the tonal contexts out of the middleground representation. We can overcome this problem without throwing the proverbial baby—the idea of deep structural voice leading—out with the bathwater by developing a more flexible theory of voice leading that can apply to structures other than fixed cardinality chords.

The DFT reparameterizes a pcset by modeling it with sinusoidal pc-distributions that divide the octave evenly into 1–11 parts. The phase of these components indicates which perfectly even distribution the pcset best approximates. The special advantage of the DFT is that it applies to any cardinality of chord [7]. We are interested here primarily in the phase of the third component, which can be interpreted as an arrangement of *triadic orbits*. These reflect triadic voice-leading properties of a pcset regardless of whether it is a triad or three-note chord. Figure 1 shows the triadic orbits of all the significant keys that appear in the *Heiliger Dankgesang* movement, with major keys represented by the intersection of their tonic and dominant seventh chords, and minor keys by the harmonic minor scale. Notes near the center of the orbits (dashed lines) are most stable, while notes near the periphery (triangle vertices) are less stable and act as neighbors to notes at the center of the orbit. (These stability conditions need to be augmented, however, by information from the fifth Fourier component, which indicates which notes are at a further remove on the circle of fifths.) A succession of pcsets (representing chords, keys, etc.) turns the triadic orbit boundaries in a direction corresponding to the overall direction of triadic voice leading.

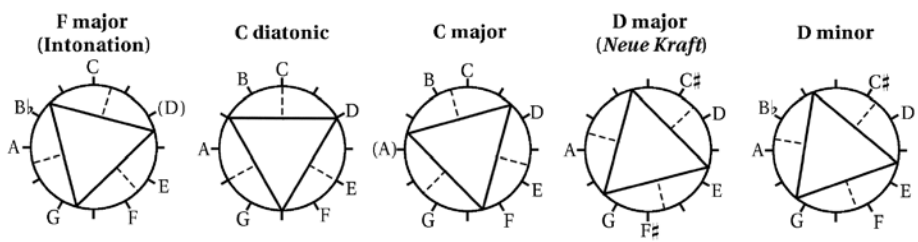

Fig. 1. Triadic orbits of keys used in the *Heiliger Dankgesang* movement.

The C-D interval is a prominent motivic element of the movement, manifest at many levels, and brought the forefront especially in the final chorale section and coda. The status of this interval constitutes one of the most significant differences between the triadic orbits of the F major tonality suggested by the initial intonation of the chorale sections, and the C major tonality established in the later phrases of the chorale. In F major, D is an upper neighbor to C, whereas in C major the notes are separated into different orbits. D remains unstable in C major, but has an upward-striving position within the triadic orbit of the tonic third. Intermediate between these states is the uninflected C diatonic scale, in which D is ambiguous between two triadic orbits.

The first phrase of the chorale tune resists outlining the F–C tetrachordal tonal space with a descent to C (Fig. 2), fulfilling this implication only in the coda, when it reappears in the bass. In the C major context of the second chorale phrase, it projects its upward-resolving character at the cadence, denying a conventional cadential move that melodically crosses a triadic orbit boundary.

The contrasting *Neue Kraft* sections of the piece juxtapose D major directly with the C major of the choral sections. These keys are polar opposites in their triadic orbits, and whereas D occupies an unstable peripheral position in C and F major, in D major it is at the center of its orbit.

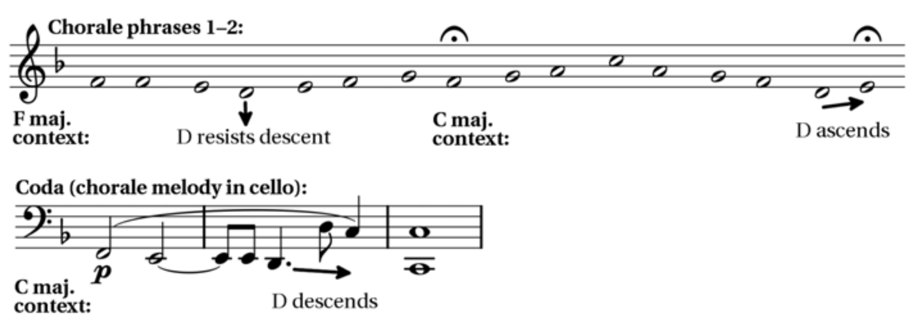

Fig. 2. Interactions of D and C in the chorale tune.

The triadic status of the note A, the tonic of the surrounding movements and the quartet as a whole, is also significant. It is stable in F major, as in A major and minor, but an upper neighbor in C major. The *Neue Kraft* section puts intense melodic focus on the A, highlighting its "new strength" as the stable center of its triadic orbit. The coda of the last chorale section recalls the *Neue Kraft* section in its repeated use of the ethereal A in the upper reaches of the first violin's range, especially in the penultimate measure, where the unstable F major chord in a C major context strives against tonality itself, towards the rarified air of the eponymous "divinity" (*gottheit*).

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"Mazzola, Galois, Riemann, Peirce and Merleau-Ponty: A Triadic, Spatial Framework for Gesture Theory"

Keywords: gesture theory, Peircean semiotics, spatial models.

Abstract:

Introduction

Guerino Mazzola has proposed a perspicuous dialectics, formed by Galois connections, or adjunctions, between formulas and gestures [2, 3, 4]. The dialectics extends his earlier, profound contributions to music theory presented in the *Topos of Music* [1], and opens up a new range of analysis, where musical interpretation *dynamizes* the complex spectrum of musical *life*. The full triadic range of sounds, partitions-formulas, and gestures becomes then suitable for complex, multilayered conceptualizations.

We can profit from earlier semiotic, philosophical and mathematical constructions to enrich Mazzola's approach. Three main lines of thought (*i-iii*) seem interesting:

(i) **PEIRCE**'s triadic sign (object-representamen-interpretant) helps to multiply, or extend continuously in space [7], dyadic polarities (such as Galois connections or adjunctions [8]). An adequate use of Peirce's triadic semeiotics should help then to expand Mazzola's multilayered conception of music. Moreover, Saint-Victor's definition of "gesture" (movement and figuration with an aim, fostered by Mazzola) is fully pragmaticist in Peirce's sense.

(ii) **MERLEAU-PONTY**'s *entrelacs* and *chiasme* [5, 6] postulate a gluing of subject/object, being/world, mind/body where the chiasma (crossing of optic nerves on the brain) helps to explain passages between visibility and non-visibility. A similar chiasmatic experience is in act in music, along the Galois connection formulas–gestures.

(iii) A long tradition in French philosophy of mathematics has acknowledged the importance of gestures in knowledge. Mazzola has reckoned [2] the importance of Merleau-Ponty, Cavaillès, Deleuze, Châtelet, Alunni, but those brief mentions may be expanded to a wider underlying philosophical corpus for gesture theory. On another hand, Mazzola's compression/unfolding functors between formulas and gestures recall the dual processes uniformization/ramification in **RIEMANN** surfaces [9], dear to many French philosophers of mathematics.

Our contribution

Profiting from *(i)-(iii)*, we will extend Mazzola's diagrams for the dialectic (Galois connection / adjunction) formulas–gestures along the following *dynamical sketch*, to be amplified to a multi-level Riemannian and Peircean mode, with, at the center, Merleau-Ponty's *entrelacs* or *chiasme*.

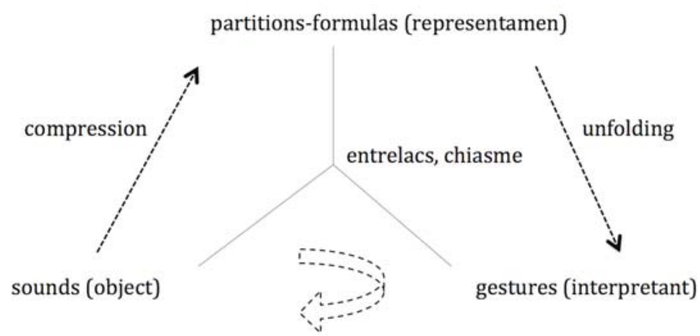

Figure 1. *The evolving diagram of musical semiosis.*

The diagram evolves in three dimensions (tetrahedron with three main triangles around the entrelacs/chiasme) producing the increasing spatial spiral (with many leaves in a Riemann surface) of musical architecture. A multilayered *transit* between the paths, faces and leaves of a complex geometrical structure should help then to explain the ambiguities, richness and variety of musical experience.

Sounds compress in formulas, which unfold in gestures, which produce sounds, which generate new musical signs and actions, continuing along Riemann's ramifications and Peirce's infinite semiosis. Merleau-Ponty's *entrelacs* enter the picture along the weaving of many forms of reflexivity between objects, representamens and interpretants.

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